



# SCMS Seminar

## Hamilton-Connected Hourglass-free Line Graphs

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**Time: Jan 6th, 10:00 - 11:00**

**Venue: Room 106, SCMS**

### **Abstract:**

Motivated by Thomassen's conjecture that every 4-connected line graph is Hamiltonian, and as line graphs are  $K_{1,3}$ -free graphs, many researchers have investigated the Hamiltonian properties of graphs forbidding certain induced graphs including  $K_{1,3}$ . The hourglass  $\Gamma_0$  is the unique simple graph with degree sequence  $(4, 2, 2, 2, 2)$  and  $P_n$  is the path on  $n$ . In [Discrete Mathematics 341 (2018) 1806-1815], Z. Ryjáček, P. Vrána and L. Xiong posed a conjecture that every 3-connected  $\{K_{1,3}, \Gamma_0, P_{16}\}$ -free graph is Hamilton-connected. X. Liu and L. Xiong in [Discrete Mathematics 345(2022), 112910] proved this conjecture. We continue to study this problem aiming to characterize all extremal graphs. We have found a family  $\mathcal{W}$  of graphs formed by subdividing the Wagner graph and by attaching pendent vertices, and prove that every 3-connected  $\{K_{1,3}, \Gamma_0, P_{18}\}$ -free graph  $G$  is Hamilton-connected unless the Hamilton-connected closure of  $G$  is a member of  $\mathcal{W}$ .