Recent Progress on Diophantine Tuples over Finite Fields

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Time: Sep 18th, 14:00 - 15:00
Venue: Room 106, SCMS

Abstract:

A set $\{a_1, a_2, \ldots, a_m\}$ of distinct positive integers is called a *Dioph--antine m-tuple* if the product of any two distinct elements in the set is one less than a square. The long-standing conjecture on the non-existence of Diophantine 5-tuples was recently resolved in the affirmative, and the notion of Diophantine m-tuples has since been generalized and studied in various contexts.

In this talk, I will consider (generalized) Diophantine tuples over finite fields. Two fundamental questions arise naturally:

1. What is the largest size of Diophantine tuples in a finite field?

2. How many Diophantine tuples exist in a finite field?

I will discuss recent progress on these two questions. A particularly interesting aspect of our work is the use of the quasi-randomness of (hyper)graphs, which is a central theme in extremal combinatorics. I will explain how combinatorial methods can be applied to study these number-theoretic questions, and also describe how we used Stepanov's method in our analysis.

Most of the results I will present are based on joint work with Seoyoung Kim (University of Basel) and Chi Hoi Yip (Georgia Institute of Technology).