COHOMOLOGY, HYPERBOLIC GROUPS, DEHN FILLING AND SMALL CANCELLATION

Speaker: Bin Sun
Michigan State University

Time: Wed, May 28th, June 4th, June 11th, 15:00-17:00pm
Fri, May 30th, 14:00-16:00pm; June 6th, June 13th, 15:00-17:00pm

Venue: Room 102, SCMS
Zoom meeting ID: 646 617 8889 Passcode: 123456wu

Abstract:

A question of Talelli asks whether there exists a torsion-free group G with $\mathrm{cd}(G) = \infty$, such that there exists a constant k with the property that every subgroup $H < G$ with $\mathrm{cd}(H) < \infty$ in fact satisfies $\mathrm{cd}(H) \leq k$, where cd stands for the cohomological dimension of a group. I will talk about recent joint work with Francesco Fournier-Facio where we answered this question in the affirma tive. More precisely, we constructed a torsion-free *Tarski monster* G; i.e., G is non-abelian, and non-trivial proper subgroups of G are all isomorphic to Z; such that $\mathrm{cd}(G) = \infty$.

The construction of Tarski monsters was first done by Alexander Olshanskii by developing his small cancellation theory on hyperbolic groups. I will first define hyperbolic groups and discuss the classical small cancellation theory for free groups, and then talk about Olshanskii's small cancellation theory and his construction of Tarski monsters.

In order to obtain a Tarski monster with infinite cohomological dimension, we combine small cancellation theory with Dehn filling, which is a quotienting process applied mostly to relatively hyperbolic groups, a generalization of hy perbolic groups. I will define relatively hyperbolic groups and discuss the small cancellation theory for them. I will then define Dehn filling, and explain how it provides control on cohomology. Finally, I will construct a Tarski monster with additional control on its cohomological dimension. This short course will be accessible to graduate students.