Bound on Shortest Cycle Covers

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Abstract:

Assume G is a bridgeless graph. A cycle cover of G is a collection of cycles of G such that each edge of G is contained in at least one of the cycles. The length of a cycle cover of G is the sum of the lengths of the cycles in the cover. The minimum length of a cycle cover of G is denoted by $cc(G)$. The minimum length of a cycle cover of G consisting of k cycles is denoted by $cc_k(G)$. It was proved independently by Alon and Tarsi and by Bermond, Jackson, and Jaeger that $cc(G) \leq \frac{5}{3}m$ for every bridgeless graph G with m edges. This remained the best-known upper bound for $cc(G)$ for 40 years. In this paper, we prove that if G is a bridgeless graph with m edges and n_2 vertices of degree 2, then $cc_3(G) < \frac{29}{18}m + \frac{1}{18}n_2$. As a consequence, we show that $cc_3(G) \leq \frac{5}{3}m - \frac{1}{42}\log m$. The upper bound $cc(G) < \frac{29}{18}m \approx 1.6111m$ for bridgeless graphs G of minimum degree at least 3 improves the previous known upper bound $1.6258m$. A key lemma used in the proof confirms Fan's conjecture that if C is a circuit of G and G/C admits a nowhere zero 4-flow, then G admits a 4-flow f such that $E(G) - E(C) \subseteq \text{supp}(f)$ and $|\text{supp}(f) \cap E(C)| > \frac{3}{4}|E(C)|$.

This is a joint work with Deping Song.