

Twisted homology jump loci, twisted Alexander polynomials, and Sigma-invariants

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Abstract:

The twisted Alexander polynomials of a space, associated to a linear representation of the fundamental group, are non-abelian refinements of the classical Alexander polynomial from knot theory. In this paper, we show that they arise naturally from a new family of invariants---the twisted homology jump loci---which extend the rank-one characteristic varieties to higher-rank local systems. Using the tropical geometry of these twisted loci, we obtain sharper upper bounds for the Bieri--Neumann--Strebel--Renz (BNSR) Sigma-invariants.

For compact orientable 3-manifolds with toroidal or empty boundary, we use a theorem of Friedl--Vidussi to show that the closure of the union of these twisted tropical bound is sharp: it recovers the fibered faces of the Thurston norm ball exactly, a result that fails without twisting. For compact Kahler manifolds, we prove that the twisted Alexander polynomial must equal 0 or 1. Both results provide obstructions to geometric realizability that are strictly stronger than their classical untwisted counterparts. Joint work with A.I. Suciu.