# Week 2 (Sep.18-Sep.22)

**Topic:** Incompressible Navier-Stokes Equations

#### Workshop Room: Room 2201, Guanghua East Building, Fudan University

Lecture Series Speakers: Hideo Kozono (Waseda University) Fanghua Lin (Courant Institute, NYU)

Invited Speakers: Guilong Gui (Northwest University) Bin Han (Hangzhou Dianzi University) Ning Jiang (Wuhan University) Shangkun Weng (Wuhan University) Qi Zhang (University of California Riverside) Zhifei Zhang (Peking University)

#### **Organizing Committee:**

Peter Constantin (Princeton University) Yoshikazu Giga (University of Tokyo) Hao Jia (University of Chicago) Carlos Kenig (University of Chicago) Zhen Lei (Fudan University) Fanghua Lin (Courant Institute of Mathematical Sciences) Gregory Seregin (University of Oxford) Vladimir Sverak (University of Minnesota) Edriss Titi (Texas A & M University) Sijue Wu (University of Michigan)

#### Sponsored by

Shanghai Center for Mathematical Sciences School of Mathematical Sciences, Fudan University

#### For further information, please contact

Ke Han (hanke@fudan.edu.cn) Zhen Lei (zlei@fudan.edu.cn)

Schedule

# 2017 Fall Program on Analysis of PDE (Sept. 11 – Dec. 2, 2017)

Week 2 (Sept.18-Sept.22)		
Topic: Incompressible Navier-Stokes Equations		
Monday (September 18) Room 2201, Guanghua East Building, Fudan University		
Morning Session		
9:30 – 9:35	Chair: Fanghua Lin	
9:35 – 10:25	Hideo Kozono (Lecture 1)	
10:25 - 10:45	Tea Break	
10:45 - 10:50	Chair: Hideo Kozono	
10:50 - 11:40	Ning Jiang	
Lunch Break		
Afternoon Session		
14:30 - 14:35	Chair: Ning Jiang	
14:35 – 15:25	Fanghua Lin (Lecture 1)	
15:25 – 15:45	Tea Break	
15:45 – 15:50	Chair: -	
15:50 - 16:40	Free	
Tuesday (September 19) Room 2201, Guanghua East Building, Fudan University		
Morning Session		
9:30 – 9:35	Chair: Fanghua Lin	
9:35 – 10:25	Hideo Kozono (Lecture 2)	
10:25 - 10:45	Tea Break	
10:45 - 10:50	Chair: Hao Wu	
10:50 - 11:40	Qi Zhang	
Lunch Break		

Afternoon Session	
14:30 - 14:35	Chair: Qi Zhang
14:35 – 15:25	Fanghua Lin (Lecture 2)
15:25 - 15:45	Tea Break
Group Photo	15:45 – 16:00
16:00 - 16:05	Chair: Qi Zhang
16:05 - 16:55	Fanghua Lin (Lecture 3)
Wednesday (Septemb	er 20) Room 2201, Guanghua East Building, Fudan University
Morning Session	
9:30 - 9:35	Chair: Fanghua Lin
9:35 - 10:25	Hideo Kozono (Lecture 3)
10:25 - 10:45	Tea Break
10:45 - 10:50	Chair: Hideo Kozono
10:50 - 11:40	Bin Han
Lunch Break	
Afternoon Session	
14:30 - 14:35	Chair: Qi Zhang
14:35 – 15:25	Fanghua Lin (Lecture 4)
15:25 - 15:45	Tea Break
15:45 - 15:50	Chair: Qi Zhang
15:50 - 16:40	Fanghua Lin (Lecture 5)
Thursday (Septembe	er 21) Room 2201, Guanghua East Building, Fudan University
Morning Session	
9:30 – 9:35	Chair: Fanghua Lin
9:35 – 10:25	Hideo Kozono (Lecture 4)
10:25 - 10:45	Tea Break
10:45 - 10:50	Chair: Hideo Kozono
10:50 - 11:40	Zhifei Zhang
Lunch Break	

Afternoon Session			
14:30 - 14:35	Chair: Zhifei Zhang		
14:35 – 15:25	Fanghua Lin (Lecture 6)		
15:25 – 15:45	Tea Break		
15:45 – 15:50	Chair: Zhifei Zhang		
15:50 - 16:40	Guilong Gui		
Friday (September 22) Room 2201, Guanghua East Building, Fudan University			
Morning Session			
9:30 – 9:35	Chair: Fanghua Lin		
9:35 – 10:25	Hideo Kozono (Lecture 5)		
10:25 - 10:45	Tea Break		
10:45 - 10:50	Chair: Zhen Lei		
10:50 - 11:40	Shangkun Weng		
Lunch Break	Lunch Break		
Afternoon Session			
14:30 - 14:35	Chair: -		
14:35 – 15:25	Free		
15:25 – 15:45	Tea Break		
15:45 – 15:50	Chair: -		
15:50 - 16:40	Free		

#### 2017 Fall Program on Analysis of PDE

Week 2 (Sep.18-Sep.22)

**Topic:** Incompressible Navier-Stokes Equations

#### **Titles and Abstracts:**

#### **Speaker: Guilong Gui**

# Title: Global well-posedness of the 2-D incompressible Navier-Stokes-Cahn-Hilliard system with a singular free energy density

**Abstract:** In this talk, we are concerned with the subject of the 2-D incompressible Navier-Stokes-Cahn-Hilliard (NS-CH) system with a singular free energy density, which is a diffuse interface model, and describes the evolution of droplet formation and collision during flow of viscous incompressible Newtonian fluids of the same density but different viscosity. Due to lack of the maximum principle for the Cahn-Hilliard equation (as a fourth-order parabolic equation), we construct its approximate second parabolic equation, and use comparison principle and the basic energy estimates to separate the solution from the singular values of the singular free energy density, where the comparison principle and Orlicz embedding theorem play a key role. Based on these, we prove the global well-posedness of the Cauchy problem of the 2-D NS-CH equations with periodic domain by using the energy estimate with the Logarithmic Sobolev inequality.

#### Speaker: Bin Han

#### Title: Sharp one component regularity for Navier-Stokes

Abstract: We consider the conditional regularity of mild solution v to the incompressible Navier-Stokes equations in three dimensions. Let  $e \in in \mathbb{S}^2$  and  $0 < T^{ast} < \inf v$ . J. Chemin and P. Zhang (Ann. Sci. 'E. Norm. Sup'e, 2016) proved the regularity of v on  $(0,T^{ast})$  if there exists  $p \in (4, 6)$  such that  $\inf_0^T \otimes i \in \mathbb{N}^2 \otimes i \in \mathbb{N}^2$ . Chemin, P. Zhang and Z. F. Zhang (Arch. Ration. Mech. Anal., 2017) extended the range of  $p \in (4, \inf v)$ . In this article we settle the case  $p \in [2, 4]$ . Our proof also works for the case  $p \in (4, \inf v)$ .

#### **Speaker: Ning Jiang**

#### Title: Global smooth solutions to the incompressible Navier-Stokes-Maxwell equations

**Abstract:** We prove the global in time smooth solutions to the incompressible Navier-Stokes-Maxwell equations with small initial data. We employ the linear wave equation with damping the magnetic field B which was not used in the previous works of Ibrahim, Masmoudi and Germain. In this talk I also will outline

how to derive these equations from Vlasov-Maxwell-Boltzmann equations. This is a joint work with Yilong Luo.

#### Speaker: Hideo Kozono

# Title: Method of the Besov space and its applications to the strong solutions of the Navier-Stokes equations

**Abstract:** We first introduce several basic notions of the Besov spaces such as paraproduct formula and the chain rule. The bilinear estimates related to the nonlinear structure on the Navier-Stokes equations and the \$L^p-L^q\$-type estimates of the Stokes semigroup are established. Then the problem on existence, uniqueness and regularity of the stationary Navier-Stokes equations is discussed in the scaling invariant homogeneous Besov space. In particular, a self-similar solution is constructed. As for the non-stationary case, we prove a maximal regularity theorem of the Stokes equations in the homogeneous Besov space. Finally, local and global well-posedness in the critical Besov space is fully investigated for the Cauchy problem with the external forces of the non-stationary Navier-Stokes equations.

This series of lectures is based on the joint work with Prof. Senjo Shimizu at Kyoto University.

#### **Speaker: Fanghua Lin**

#### **Title: Regularity Theory of the Navier-Stokes Equations**

**Abstract:** We first give a unified approach to several classical regularity theorems for the Navier- Stokes equations using the dimension analysis and the blow up method. These arguments arrive at end the Caffarelli-Kohn-Nirenberg partial regularity theory and its elliptic counterpart for 5D-stationary Navier Stokes equations. Our main focus would be to discuss recent works by Seregin, Sverak and others concerning \$L^infty (L^3)\$ solutions of NS equations. As some of these works are relatively recent and technical, we invite audience to participate to put forward their thoughts and to join in various discussions.

#### Speaker: Shangkun Weng

**Title: Decay properties of axially symmetric Leray solutions to the steady Navier-Stokes equations Abstract:** In this talk, I first present some Liouville type theorems for axially symmetric D-solutions to steady Navier-Stokes equations. I will also talk about our recent results on the decay properties of smooth axially symmetric D-solutions to the steady Navier-Stokes equations with an arbitrary large forcing term.

## Speaker: Qi Zhang Title: Regularity of Weak Solutions of Elliptic and Parabolic Equations with Some Critical or Supercritical Potentials

**Abstract:** We prove Holder continuity or differentiability of weak solutions of uniformly elliptic and parabolic equations with power like potentials which scales critically or super-critically. This is a joint work with Li Zijin.

## Speaker: Zhifei Zhang Title: Well-posedness of Prandtl equation and inviscid limit of the Navier-Stokes equations

### **Participants:**

Dongfen Bian (Beijing Institute of Technology) Yuan Cai (Fudan University) Tuowei Chen (Fudan University) Xiufang Cui (Fudan University) Guilong Gui (Northwest University) Bin Han (Hangzhou Dianzi University) Bobo Hua (Fudan University) Ning Jiang (Wuhan University) Zhentao Jin (Fudan University) Hideo Kozono (Waseda University) Zhen Lei (Fudan University) Hui Li (Peking University) Jingyue Li (China Academy of Engineering Physics) Fanghua Lin (Courant Institute) Junren Luo (Fudan University) Xiang Luo (University of Science and Technology of China) Jianzhong Min (Fudan University) Yao Nie (Beihang University) Yun Pu (Fudan University) Peng Qu (Fudan University) Jiawei Sun (Capital Normal University) Houzhi Tang (Capital Normal University) Chenmu Wang (Fudan University) Yanyan Wang (Fudan University) Shangkun Weng (Wuhan University) Hao Wu (Fudan University) Xiaochun Wu (Fudan University) Jing Zhang (Fudan University)

Qi Zhang (University of California, Riverside) Zhifei Zhang (Peking University) Qidi Zhang (Chinese Academy of Sciences) Na Zhao (Fudan University) Yi Zhou (Fudan University)