

Week II Workshop on Moduli Spaces and Related Problems

Monday, 9.17

E.J.N. Looijenga

Title: Moduli of Riemann surfaces with prescribed automorphism group.

Time: 9:30--10:30

Abstract: Riemann surfaces endowed with a faithful action of a finite group of a given topological type have their own moduli space. These have monodromy representations which commute with the action of the finite group and give rise to virtual representations of the mapping class group of the orbit surface. With the help of algebraic geometry of curves and Hodge theory, we obtain some properties of these (topologically defined) representations.

Qile Chen

Title: Witten's r-spin class via logarithmic compactification

Time: 11:00--12:00

Abstract: FJRW theory/Witten's top Chern classes were constructed by Fan-Jarvis-Ruan using analytic methods. An algebraic approach has been developed by Chang-Li-Li and Chang-Kiem-Li using Kiem-Li cosection localization. They proved that the corresponding virtual cycle is represented by a Chow cycle supported on a proper locus inside a non-proper moduli space parameterizing holomorphic sections of certain line bundles. In this talk, I will focus on the r-spin example to introduce a logarithmic compactification using stable logarithmic maps of Abramovich-Chen-Gross-Siebert. This new proper moduli space carries a reduced perfect obstruction theory whose associated virtual cycle is Witten's r-spin class. This is a step toward a localization formula for higher genus invariants conjectured by Felix Janda. This is a joint work with Felix Janda, Yongbin Ruan, and Adrien Sauvaget

Martin Olsson

Title: On Fundamental groups of degenerating varieties.

Time: 14:00--15:00

Abstract: I will discuss a version of Grothendieck's specialization theorem for fundamental groups in the logarithmic setting. Applications include $\text{\`{e}tale}$ versions of results of Nakayama and Ogus on variation of fundamental groups.

Zhiyu Tian

Title: Topology of spaces of rational curves

Time: 15:30--16:30

Abstract: I will explain a Floer type heuristic from symplectic geometry about the homotopy types of moduli spaces of rational curves on a rationally connected variety (due to Cohen, Jones, and Segal), and its connection with Chow groups and counting rational curves over finite fields.

Tuesday, 9.18

Burt Totaro

Title: Bott vanishing

Time: 9:30--10:30

Abstract: Bott proved a very strong vanishing theorem for sheaf cohomology on projective space. The statement does not hold for most varieties, and we survey what is known. We find new varieties that satisfy Bott vanishing, building on our knowledge of moduli spaces of K3 surfaces.

Dragos Oprea

Title: Tautological integrals over the Hilbert scheme of points

Time: 11:00--12:00

Abstract: Recently there has been progress in evaluating the generating series of Segre integrals of tautological vector bundles over the Hilbert schemes of points on surfaces. In rank 1, the Segre series were the subject of a conjecture of Lehn from 1999. Quite surprisingly, in arbitrary rank, the Segre series are also conjecturally related to the Verlinde-type generating series of Euler characteristics of tautological line bundles over the Hilbert scheme. I will explain this connection in arbitrary rank. Along the way, I will explain a conjectural extension of Lehn's formula from rank 1 to rank 2. This is based on joint work with Alina Marian and Rahul Pandharipande.

Emanuele Macri

Title: Derived categories of cubic fourfolds and non-commutative K3 surfaces

Time: 11:00--12:00

Abstract: The derived category of coherent sheaves on a cubic fourfold has a subcategory which can be thought as the derived category of a non-commutative K3 surface. This subcategory was studied recently in the work of Kuznetsov and Addington-Thomas, among others. In this talk, I will present joint work in progress with Bayer, Lahoz, Nuer, Perry, Stellari, on how to construct Bridgeland stability conditions on this subcategory. This proves a conjecture by Huybrechts, and it allows to start developing the moduli theory of semistable objects in these categories, in an analogue way as for the classical Mukai theory for (commutative) K3 surfaces. I will also discuss a few applications of these results.

Dan Petersen

Title: Tautological classes with twisted coefficients

Time: 15:30--16:30

Abstract: Richard Hain has defined an "enlarged" tautological ring, which is a subalgebra of the cohomology ring of the moduli space of smooth curves M_g with coefficients in the algebraic coordinate ring of the symplectic group. In joint work with Tavakol and Yin we calculated this ring completely in genus up to 4, and obtained partial results also in higher genera. Understanding this ring in fixed genus is equivalent to understanding all tautological rings of all fibered powers of the universal curve over M_g simultaneously. A conjecture of Yin regarding tautological classes on the universal jacobian variety is equivalent to the claim that this enlarged tautological ring is isomorphic to the exterior algebra on the symplectic representation V_{111} modulo the ideal generated by the subrepresentation V_{22} occurring in the exterior square

Junliang Shen

Title: Perverse filtrations, Hilbert schemes, and the P=W conjecture.

Time: 16:45--17:45

Abstract: For a smooth surface fibered over a curve, the Hilbert scheme of n points on the surface admits a filtration on the rational cohomology groups coming from the perverse t-structure. I will discuss how to locate the tautological classes on the Hilbert scheme of points with respect to the perverse filtration. Then I will explain the applications of our results to the universal classes of certain moduli spaces of parabolic Higgs bundles constructed by Gorsky-Nekrasov-Rubtsov and Grochenig. The connection to the P=W conjecture will be discussed. Based on the joint work

(in progress) with ZiLi Zhang.

Wednesday, 9.19

Valery Alexeev

Title: Reflection groups and compact moduli Time: 9:30--10:30

Time: 9:30--10:30

Abstract: I will discuss a series of compactifications of moduli spaces of surfaces associated with reflection groups. Based on a joint work with Alan Thompson.

Georg Oberdieck

Title: TBA

Time: 11:00--12:00

Kang Zuo

Title: Higgs Bundles and Hyperbolicities

Time: 14:00--15:00

Abstract: The theory of period maps has been powerful in the study of higher dimensional Shafarevich program under the assumption of the injectivity of Kodaira-Spencer deformation on Hodge bundles. Viehweg-Zuo constructed a class of Higgs bundles on moduli spaces by combining Kodaira-Spencer deformation and Hodge theory, which is accessible in the most general situation, where period maps fail being locally injective.

More explicitly, for a family $f : X \rightarrow Y$ parametrizing n -folds F of semi ample ω_F and with the degeneration over a closed subvariety $D \subset Y$, Viehweg-Zuo introduced a Higgs bundle (G, τ) over Y with singularities over D by extending the Kodaira-Spencer deformation \mathcal{T} on the higher order cohomologies of the tangent bundle along F . The central feature of this Higgs bundle is that there exists a natural comparison map $\rho : (G, \tau) \rightarrow (E, \theta)$, where (E, θ) is the graded Higgs bundle of the variation of Hodge structures of the relative middle cohomology on the cyclic cover $g : Z_s \rightarrow X \rightarrow Y$ defined by a section from the linear system of the relative pluri-canonical line bundle on X twisting a small anti ample line bundle on Y . The Hodge metric on $\ker(\theta)$ becomes a non-zero (possibly degenerated) negatively curved Finsler metric on $Y \setminus D =: U$ via the iterated Kodaira-Spencer deformation and if the second graded piece $\rho^{n-1,1}$ of ρ is injective on T_U . This Finsler metric plays a crucial role in the study of hyperbolicities of U by many people. Indeed $\rho^{n-1,1}$ holds being injective for two extreme cases $\kappa(F) = n$ and $\kappa(F) = 0$ by Bogomolov vanishing theorem and the trivial reason. For the general case $0 \leq \kappa(F) \leq n$ Viehweg-Zuo showed that it is generically injective along any algebraic curve in U . Very recently X.Lu, R.R. Sun and K. Zuo showed $\rho_{T_U}^{n-1,1}$ is injective for $\kappa(F) = 1$ by investigating Iitaka fibration.

Besides Brody, Kobayashi and Viehweg hyperbolicities we rise a conjecture on Borel and topologic hyperbolicities. If time permits I shall outline an approach towards to the conjecture.

This is a joint project with A. Javanpeykar, X. Lu, and R.R. Sun.

Kuan-Wen Lai

Title: Derived equivalences of K3 surfaces via Cremona transformations

Time: 15:15--16:15

Abstract: It is well-known that the derived equivalences of K3 surfaces can be realized as the isometries between the Mukai lattices. Initiated by the study of the rationality problem of cubic fourfolds, we found that such equivalences are explained via the birational automorphisms of \mathbb{P}^4 for general degree 12 K3 surfaces. This talk will mainly be devoted to the construction of the examples. As a corollary, I will exhibit an interesting relation in the Grothendieck ring of complex algebraic varieties which can be derived easily from the construction.

Thursday, 9.20

Bumsig Kim

Title: Fundamental Factorization of a GLSM

Time: 9:30--10:30

Abstract: A Landau-Ginzburg model is a pair of a smooth stack and a regular function on it. For a Landau-Ginzburg model one would like to have a Gromov-Witten type theory: a certain cohomological field theory counting curves in the model. There are some developments when the model is a gauged linear sigma model. We discuss the notion of "fundamental factorization" on the moduli space of stable Landau-Ginzburg maps, which plays the role of virtual fundamental classes in derived level. Polishchuk and Vaintrob constructed fundamental matrix factorizations for pure LG models to define Fan-Jarvis-Ruan-Witten theory algebraically. We generalize their construction to hybrid models.

This talk is based on joint work with Ciocan-Fontanine, Favero, Guere, and Shoemaker

Huai-Liang Chang

Title: Application of Mixed Spin P field theory.

Time: 11:00--12:00

Abstract: Mixed Spin P field is designed to study Gromov Witten invariants of Calabi Yau threefolds. It determines reduce the enumeration of GW invariants to finite many initial datas. I will report on its progress.

Jochen Heinloth

Title: One-Parameter subgroups in moduli problems and applications.

Time: 14:00--15:00

Abstract: Recently Alper, Hall and Rydh gave general criteria when a moduli problem can locally be described as a quotient and thereby clarified the local structure of algebraic stacks.

We report on a joint project with Jarod Alper and Daniel Halpern-Leistner in which we use these results to show general existence and completeness results for good coarse moduli spaces.

In the talk we will focus on two aspects that illustrate how the geometry of algebraic stacks gives a new point of view on classical methods for the construction of moduli spaces. Namely we explain how one-parameter subgroups in automorphism groups allow to formulate a version of Hilbert-Mumford stability in stacks that are not global quotients and sketch how one can reformulate Langton's proof of semistable reduction for coherent sheaves in geometric terms. This allows to apply the method to an interesting class of moduli problems.

Young-Hoon Kiem

Title: Cosection localization and quantum singularity theory

Time: 15:30--16:30

Abstract: I will discuss a recent joint work (arXiv:1806.00116) with Jun Li in which we generalize the cosection localization to intersection homology and Borel-Moore homology that provides us with a purely topological construction of FJRW invariants and some GLSM invariants for both broad and narrow sectors.

XiaoWei Wang

Title: Compact moduli space of Fano Kähler-Einstein varieties

Time: 16:45--17:45

Abstract: In this talk, we will discuss our construction of proper schemes as moduli spaces parametrizing smoothable K-stable Fano varieties. The solution relies on the recent solution of the Yau-Tian-Donaldson conjecture by Chen-Donaldson-Sun and Tian. (This is a joint work with Chi Li and Chenyang Xu)

Friday, 9.21

Kieran O'Grady

Title: Degenerations and periods of Debarre-Voisin varieties.

Time: 9:30--10:30

Abstract: Given a trilinear alternating form σ on a 10-dimensional complex vector space V_{10} , one may consider the set K_σ of 6 dimensional subspaces of V_{10} on which σ restricts to 0. Debarre and Voisin proved that if σ is generic, then K_σ is a hyperkahler (HK) variety of Type $K3^{[2]}$, and that by varying σ one gets a locally complete family of polarized HK's (the polarization has degree 22 and divisibility 2 in the BBF lattice). I will report on joint work with O.~Debarre, F.~Han and C.Voisin, in which we analyze a generic degeneration of D-V varieties, K_{σ_t} ,

when σ_t approaches certain very special trivectors with large stabilizer group. The limit D-V variety K^{al} is not a HK (usually it is of dimension greater than 4). We prove that the central fiber can be replaced by a suitable HK birational to a Hilbert square of a K3, and that the corresponding periods approach the generic point of certain Heegner divisors in the period space. An analogue in the family of varieties of lines on a cubic fourfold is the generic degeneration of a cubic to the discriminant cubic.

Benjamin Bakker

Title: o-minimal GAGA and applications to Hodge theory

Time: 11:00--12:00

Abstract: Hodge structures on cohomology groups are fundamental invariants of algebraic varieties; they are parametrized by quotients D/Γ of period domains by arithmetic groups. Except for a few very special cases, such quotients are never algebraic varieties, and this leads to many difficulties in the general theory. We explain how to partially remedy this situation by equipping D/Γ with an o-minimal structure in which any period map is definable. The algebraicity of Hodge loci is an immediate consequence via a theorem of Peterzil--Starchenko. We further prove a general GAGA type theorem in the definable category, and deduce some finer algebraization results. This is joint work with Y. Brunebarbe, B. Klingler, and J. Tsimerman.