Yusuke Nakamura

Title: A rational point problem on Fano varieties.

Abstract: Fano varieties defined over a certain field k are believed to have a k-rational point. For instance, the classically known Chevalley--Warning theorem states that homogeneous polynomials $f_1, \ldots, f_l \in \mathbb{F}_q [x_0, \ldots, x_n]$ with $(n+1)$-many variables over a finite field $\mathbb{F}_q$ have a non-trivial common solution if $\sum_{1 \le i \le l} \deg f_i \le n$ holds. More precisely, it states that the number of common solution is divisible by the characteristic p of the field $\mathbb{F}_q$. Geometrically, it can be interpreted as that the number of the $\mathbb{F}_q$-rational points on a complete intersection Fano variety is one modulo p. This kind of problem was answered in the smooth case by Esnault. She proved that smooth Fano varieties defined over a finite field have a rational point. It is natural to ask whether we can generalize this results to singular Fano varieties or not. In this talk, we study the rational point problem on singular Fano varieties and the relation to the minimal model program.

Johannes Nicaise

Title: The non-archimedean SYZ fibration and Igusa zeta functions

Abstract: The Strominger-Yau-Zaslow conjecture gives a geometric description of the relation between mirror pairs of Calabi-Yau varieties. It was a fundamental insight of Kontsevich and Soibelman that the structures predicted by the SYZ conjecture can be found in the world of non-archimedean geometry (Berkovich spaces). I will explain some of the main ideas, as well as the connections with the minimal model program in birational geometry. I will also discuss how these results have led to a proof of Veys's conjecture on poles of maximal order of Igusa's p-adic zeta functions. This is based on joint work with Mircea Mustata, Chenyang Xu and Tony Yue Yu.

Karl Schwede

1. Title: Measuring singularities via resolution or with Frobenius
Abstract: In this talk I will discuss the relationship between singularities in characteristic zero coming from the minimal model program and in characteristic p >

0. Specifically, F-rational, F-regular, F-pure and F-injective singularities will appear, as well as test ideals.

2. Title: Global applications of Frobenius
Abstract: I will demonstrate ways in which Frobenius can be used to replace Kawamata-Viehweg vanishing theorems in some simple applications, and how the singularities from talk #1 naturally play a role.

3. Title: Mixed characteristic singularities via perfectoid big Cohen-Macaulay algebras
Abstract: I will discuss recent work with Linquan Ma which uses perfectoid big Cohen-Macaulay algebras (as recently shown to exist by Andre) as a mixed characteristic replacement for resolution of singularities or Frobenius. Some applications will also be discussed.

Zhiyu Tian

Title：Fano varieties in positive characteristic

Abstract: I will discuss some aspects of the geometry of Fano varieties in positive characteristic.
Lecture 1: Bend-and-break. I will explain Mori's celebrated proof that Fano varieties are uniruled.
Lecture 2: Separable rational connectedness and some constructions. I will discuss the question of whether Fano varieties are separably rationally connected and some constructions due to Kollár and Mingmin Shen.
Lecture 3: Fundamental group of separably rationally connected varieties. I will explain a theorem of Kollár and its applications.

Junyi Xie

1. Title: The geometric Bogomolov conjecture

Abstract: With Cantat, Habegger and Gao, we prove the geometric Bogomolov conjecture over a function field of characteristic zero. This generazed the recent work of Habegger and Gao, who proved the geometric Bogomolov conjecture over a function field of a curve.

2. Title: Invariant pencils for polynomial self-maps of the affine plan

Abstract: With Jonsson and Wulcan, we classify polynomial self-maps f of the affine plane of that preserve an irreducible pencil of curves at infinity. More generally, we study a more general classication problem, where the invariant pencil is replaced by more general numerical data at infinity.

3. Title: Algebraic dynamics of the lifts of Frobenius

Abstract: We study the algbraic dynamics for endomorphisms of projective spaces with coefficients in a p-adic field whose reduction in positive characteritic is the Frobenius. In particular, we prove a version of the dynamical Manin-Mumford conjecture and the dynamical Mordell-Lang conjecture for the coherent backward orbits for such endomorphisms. We also give a new proof of a dynamical version of the Tate-Voloch conjecture in this case. Our method is based on the theory of perfectoid spaces introduced by P. Scholze.

Lei Zhang

Title: Positivity in characteristic $p>0$

Abstract: In this lecture we will discuss the positivity of the sheaf $f_*\omega_{X/Y}^n$ for a fibration $f: X\to Y$ in positive characteristic $p>0$. Recent progresses are made by Patakfalvi, Ejiri and many others. Ejiri introduces a new invariant $t(X, \mathcal{F}, H)$ to describe how positive a sheaf $\mathcal{F}$ is, this invariant is defined via Frobenius pullback and more reasonable in characteristic p.

In particular, $t (X,\mathcal{F}, H) \geq 0$ implies the weak positivity of $\mathcal{F}$. Using this criterion, weak positivity follows easily from applying trace maps of relative Frobenius maps.

We plan to (1) discuss relative results on weak positivity in characteristic zero, and recent analogous results in characteristic p, (2) recall basic knowledge on Frobenius maps and trace maps, (3) explain Ejiri's proof in details and introduce one positivity results without singularity conditions,(4) show the applications in subadditivity of Kodaira dimensions.