Week 12 (Nov.27 - Dec.1)

Topic: MHD and QG

Workshop Room: Room 2201, Guanghua East Building, Fudan University

Lecture Series Speakers:

Edriss Titi (Texas A&M University and The Weizmann Institute of Science)

Invited Speakers:

Xuming Gu (Shanghai University of Finance and Economics)
Jinkai Li (Chinese University of Hong Kong)
Ting Zhang (Zhejiang University)

Organizing Committee:

Peter Constantin (Princeton University)
Yoshikazu Giga (University of Tokyo)
Hao Jia (University of Chicago)
Carlos Kenig (University of Chicago)
Zhen Lei (Fudan University)
Fanghua Lin (Courant Institute of Mathematical Sciences)
Gregory Seregin (University of Oxford)
Vladimir Sverak (University of Minnesota)
Edriss Titi (Texas A & M University)
Sijue Wu (University of Michigan)

Sponsored by

Shanghai Center for Mathematical Sciences
School of Mathematical Sciences, Fudan University

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Schedule

2017 Fall Program on Analysis of PDE (Sept. 11 – Dec. 2, 2017)

Week 12 (Nov.27-Dec.1)	
Topic: MHD and QG	
Monday (November 27)	**Room 2201, Guanghua East Building, Fudan University**
Morning Session	
9:30 – 9:35	Chair：Fanghua Lin
9:35 – 10:25	Edriss Titi
10:25 – 10:45	Tea Break
10:45 – 10:50	Chair：Fanghua Lin
10:50 – 11:40	Edriss Titi
Lunch Break	
Afternoon Session	
14:30 – 14:35	Chair: Hao Wu
14:35 – 15:25	Ting Zhang
15:25 – 15:45	Tea Break
15:45 – 15:50	Chair: Hao Wu
15:50 – 16:40	Xumin Gu
Tuesday (November 28)	**Room 2201, Guanghua East Building, Fudan University**
Morning Session	
9:30 – 9:35	Chair：Ting Zhang
9:35 – 10:25	Edriss Titi
10:25 – 10:45	Tea Break and Group Photo
10:45 – 10:50	Chair：Ting Zhang
10:50 – 11:40	Jinkai Li
Lunch Break	

Afternoon Session	
14:30 – 14:35	Chair: Edriss Titi
14:35 – 15:25	Jinkai Li
15:25 – 15:45	Tea Break
15:45 – 15:50	Chair: --
15:50 – 16:40	--
Wednesday (November 29)　　Room 2201, Guanghua East Building, Fudan University	
Morning Session	
9:30 – 9:35	Chair：--
9:35 – 10:25	--
10:25 – 10:45	Tea Break
10:45 – 10:50	Chair：--
10:50 – 11:40	--
Lunch Break	
Afternoon Session	
14:30 – 14:35	Chair: -
14:35 – 15:25	**-**
15:25 – 15:45	**-**
15:45 – 15:50	Chair: -
15:50 – 16:40	**-**
Thursday　(November 30)　　Room 2201, Guanghua East Building, Fudan University	
Morning Session	
9:30 – 9:35	Chair: --
9:35 – 10:25	--
10:25 – 10:45	Tea Break
10:45 – 10:50	Chair:--
10:50 – 11:40	--
Lunch Break	
Afternoon Session	

14:30 – 14:35	Chair: --
14:35 – 15:25	**--**
15:25 – 15:45	**--**
15:45 – 15:50	Chair: --
15:50 – 16:40	**--**
Friday (December 1) Room 2201, Guanghua East Building, Fudan University	
Morning Session	
9:30 – 9:35	Chair：--
9:35 – 10:25	--
10:25 – 10:45	Tea Break
10:45 – 10:50	Chair: --
10:50 – 11:40	--
Lunch Break	
Afternoon Session	
14:30 – 14:35	Chair: -
14:35 – 15:25	--
15:25 – 15:45	--
15:45 – 15:50	Chair: -
15:50 – 16:40	--

2017 Fall Program on Analysis of PDE

Week 12 (Nov.27 – Dec.1)

Topic: MHD and QG

Titles and Abstracts:

Speaker: Xumin Gu

Title: On the Construction of Solutions to the Free-Surface Incompressible Ideal Magnetohydrodynamic Equations

Abstract: We consider a free boundary problem for the incompressible ideal magnetohydrodynamic equations that describes the motion of the plasma in vacuum. The magnetic field is tangent along the plasma-vacuum interface. We consider two kind vacuum magnet field. First, we consider the vacuum magnet field is zero, then the total pressure of plasma vanishes along the interface.Then under the Taylor sign condition of the total pressure on the free surface, we prove the local well-posedness of the problem in Sobolev spaces. Serveral new ideas are developed to overcome the essential difficulties caused by the presence of the magnetic field and the low regularity of the free boundary. Second, we consider an axially symmetric case and the vacuum magnet field is non-trivial. Then under the generalized Taylor sign condition on the free surface, we prove the local well-posedness of the problem in Sobolev spaces. Serveral new ideas are developed to overcome the essential difficulties caused by the presence of the magnetic field and the low regularity of the free boundary.

Speaker: Jinkai Li

First Lecture:

Title: Global well-posedness of some geophysical models: the dry dynamics

Abstract: The motion of the large-scale atmospheric and oceanic flows is governed by the primitive equations (PEs), which are derived from the Navier-Stokes equations by using the Boussinesq and hydrostatic approximations. The strong horizontal turbulent mixing, which creates the horizontal eddy viscosity, leads us to consider the PEs with horizontal viscosity. It will be shown that the 3D PEs with horizontal viscosity admits a unique global strong solution, for arbitrary sufficient smooth initial data, as long as one still has the horizontal or vertical thermal diffusivity. These are joint works with Chongsheng Cao and Edriss S. Titi.

Second Lecture:

Title: Global well-posedness of some geophysical models: the moisture dynamics

Abstract: In this talk, we will present some recent mathematical results, mainly the global well-posedness and convergence of the relaxation limit, on two kinds of dynamical models for the atmosphere with moisture. In the first part of this talk, which is a joint work with Edriss S. Titi [1], we will consider a tropical atmosphere model introduced by Frierson, Majda, and Pauluis (Commum. Math. Sci. 2004); for this model, we will present the global well-posedness of strong solutions and the strong convergence of the relaxation limit, as the relaxation time ε tends to zero. It will be shown that, for both the finite-time and instantaneous-relaxation systems, the H^1 regularities on the initial data are sufficient for both the global existence and uniqueness of strong solutions, but slightly more regularities than H^1 are required for both the continuous dependence and strong convergence of the relaxation limit. In the second part of this talk, which is a joint work with Sabine

Hittmeir, Rupert Klein, and Edriss S. Titi [2], we will consider a moisture model for warm clouds used by Klein and Majda (Theor. Comput. Fluid Dyn. 2006), where the phase changes are allowed, and we will present the global well-posedness of this system.

[1] Jinkai Li; Edriss S. Titi: A tropical atmosphere model with moisture: global well-posedness and relaxation limit, Nonlinearity, 29 (2016), 2674--2714.
[2] Sabine Hittmeir; Rupert Klein; Jinkai Li; Edriss S. Titi: Global well-posedness for passively transported nonlinear moisture dynamics with phase changes, Nonlinearity 30 (2017) 3676–3718.

Speaker: Edriss Titi
First Lecture:
Title: On a new blow-up criterion for the 3D incompressible Euler and SQG equations: analysis and computations
Abstract: In this lecture we will present some mathematical results concerning certain analytical models of turbulence and inviscid regularization of hydrodynamical models. In particular, we will present the alpha-Voigt inviscid regularization and formulate a blow-up criterion in terms of its regularization. We will support our analytical results by computational study of the 3D Euler equations.
Second Lecture:
Title: Analytical study of certain oceanic and atmospheric models: global regularity results
Abstract: Taking advantage of the thinness of the oceans and of the atmosphere in comparison to the radius of earth we will introduce and motivate the derivation of various planetary scale circulation models. We will show global regularity of the viscous version of these models, in particular for the anisotropic primitive equations.
Third Lecture:
Title: Analytical study of certain oceanic and atmospheric models: finite time singularity
Abstract: In this lecture I will continue with the study introduced in the previous lecture and show the formation of finite-time singularity of the 2D and 3D inviscid primitive equations. In addition, I will introduced some tropic models with moisture.

Speaker: Ting Zhang
Title: Global solutions of modified one-dimensional Schrödinger equation

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Global solutions of modified one-dimensional Schrödinger equation

Ting Zhang
School of Mathematical Sciences, Zhejiang University, Hangzhou 310027, China

Abstract

In this talk, we consider the following modified one-dimensional Schrödinger equation

$$\begin{cases} (D_t - F(D))u = \lambda|u|^2 u, \quad t > 0, x \in \mathbb{R} \\ u(x,0) = \varepsilon u_0(x), \end{cases} \tag{1}$$

where $D_t = \frac{\partial_t}{i}$, $D = \frac{\partial_x}{i}$, $F(\xi)$ is a second order constant coefficients classical elliptic symbol, u is a complex valued function, $u : \mathbb{R} \times \mathbb{R} \to \mathbb{C}$, and $\lambda = 1$ or -1 corresponding to the defocusing or the focusing case. We assume that $F(\xi)$ is a smooth function defined on $\mathbb{R}$, $\xi \to F(\xi) \in \mathbb{R}$, satisfying

$$|F(\xi)| \le c_0(1 + |\xi|^2), \ 0 < c_1 \le F''(\xi) \le c_2, \ \text{for all } \xi \in \mathbb{R}, \tag{2}$$

for some positive constants c_i, $i = 0, 1, 2$. We denote

$$\Lambda = \{(x,\xi); x + F'(\xi) = 0\}, \ \mathcal{L} = x + tF'(D). \tag{3}$$

Since F' is strictly increasing, there is a smooth strictly concave function $\phi : \mathbb{R} \to \mathbb{R}$ such that $\Lambda = \{(x,\xi); \xi - d\phi(x)) = 0\}$. When the initial datum satisfies $u_0 \in H^1$ and $xu_0 \in L^2$, we can obtain the following results:

a) (Global well-posedness and decay) There is an element $\varepsilon_0 \in (0, 1]$ such that for any $\varepsilon \in (0, \varepsilon_0)$, the equation (1) has a unique solution $u \in C([0,\infty); L^2) \cap L^4([0,+\infty); L^\infty)$, which satisfies $\nabla u, \mathcal{L}u \in C([0,T]; L^2)$, for all $T > 0$, the pointwise estimate

$$\|u(t,\cdot)\|_{L^\infty} \le C\varepsilon(1+t)^{-\frac{1}{2}}, \tag{4}$$

and the energy bound

$$\|u(t,\cdot)\|_{L^2} \le C\varepsilon, \ \|(\mathcal{L}u)(t,\cdot)\|_{L^2} \le C\varepsilon(1+t)^{C\varepsilon^2}, \tag{5}$$

$$\|(\nabla u)(t,\cdot)\|_{L^2} \le C\varepsilon(1+t)^{C\varepsilon^2}, \tag{6}$$

where C is a positive constant depending on $\|u_0\|_{H^1}$ and $\|xu_0\|_{L^2}$.

b) (Asymptotic behavior) Let u be a solution to (1) as in part (a). Then there exists a function $U(x) \in H^{1-C\varepsilon^2}(\mathbb{R})$ such that

$$u(x,t) = \frac{\varepsilon}{\sqrt{t}} U(\frac{x}{t}) \exp\{it\omega(\frac{x}{t}) + i\lambda\varepsilon^2|U(\frac{x}{t})|^2 \ln t\} + \varepsilon(1+t)^{-\frac{3}{4}+C\varepsilon^2} r(t,x), \tag{7}$$

where $\omega(x) = xd\phi(x) + F(d\phi(x))$, $r \in O_{L^\infty}(1) \cap O_{L^2}((1+t)^{-\frac{1}{4}})$, $t \ge 1$.

Participants:

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