Some properties of (n, d, λ)-graphs and generalizations

Gu, Xiaofeng
University of West Georgia

Time: Jan 3rd, 10:00 - 11:00
Zoom meeting ID: 831 1173 5066 Password: 121323
Link: https://zoom.us/j/83111735066

Abstract:

A d-regular graph on n vertices with the second largest absolute eigenvalue at most λ is called an (n, d, λ)-graph. It is well known that an (n, d, λ)-graph for which $\lambda = \Theta(\sqrt{d})$ is a very good pseudorandom graph, behaving, in many aspects, like a truly random graph. In this talk, we present some properties of (n, d, λ)-graphs, in particular, we study graph toughness. The toughness $t(G)$ of a connected graph G is defined to be the minimum of $|S|/c(G-S)$ taken over all proper vertex subset S such that $G-S$ is disconnected, where $c(G-S)$ denotes the number of components of $G-S$. For any (n, d, λ)-graph, Alon proved that $t(G) > \frac{1}{3}(\frac{d^2}{d\lambda+\lambda^2} - 1)$, through which, he showed that for every t and g there are t-tough graphs of girth strictly greater than g, thus disproved a conjecture of Chvátal on pancyclicity in a strong sense. Brouwer independently discovered that $t(G) > \frac{d}{\lambda} - 2$, and he conjectured the lower bound can be improved slightly to $t(G) \geq \frac{d}{\lambda} - 1$. We filled the small gap and confirmed this 25-year-old conjecture. We will also discuss some generalizations and related problems.