International Conference on Inverse Problems Theory and Computation

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International Summer School of Modern Mathematics – Inverse Problems

Fudan University

24 June - 3 July 2024

Organizer

- Fudan University
  - Shanghai Centre for Mathematical Sciences
  - School of Mathematical Sciences

Co-organizer

- China Society for Industrial and Applied Mathematics
  - Activity Group on Inverse Problems and Imaging

Scientific Committee

- Gang Bao (Zhejiang)
- Gunther Uhlmann (Washington)

Organizing Committee

- Xi Chen (Fudan)
- Jin Cheng (Fudan)
- Shuai Lu (Fudan)
- Lauri Oksanen (Helsinki)
- Jian Zhai (Fudan)

Contents

Conference Site

The conference and school both will take place in the **Jiangwan campus** of Fudan University (Address: 2005 Songhu Road, Shanghai). The campus is close to 'XinJiangWanCheng' station and 'GuoFan Road' station of Metro Line 10. Please have your **ID (Chinese nationals) / Passport (foreigners)** with you to get access to the campus. The WiFi will be available on campus from 24 June to 3 July. (WiFi : eduroam, Account : inverse2024@guest, Passcode : Inverse2024)

Lecture Hall

The talks and lectures will take place in Gu Lecture Hall on the 2nd floor of the SCMS Building.

Tea

Morning and afternoon tea will be served, during the events, in the lobby on the 4th floor of the SCMS Building. Accepted posters will be displayed during the tea. **All registered participants** are welcome.

Lunch

Lunch will be provided for **all registered participants** during the events in the new university cafeteria.

- Every **registered regular participant** will receive 10 lunch tickets, which are valid across the **new university cafeteria (excluding the Halal place on the 1st floor, the buffet and Peking duck places on the 3rd floor)** from 24 June to July 3 2024.

Figure 1: Jiangwan Campus

Figure 2: SCMS Building

- **Speakers and invited participants** are welcome to join the lunch in the rooms 305-307 on the 3rd floor of the new university cafeteria.

Conference Dinner

We invite **speakers and invited participants** to the conference dinner at 18:00 pm on 27 June (THU) at RuiTaiHe in the Ufun Shopping Mall (Address : 1388 Yinhang Road, 3rd floor, near 'XinJiangWanCheng' Station).

Accommodation

- We have reserved rooms for **speakers and invited participants** at Fraser Place (Address : 1258 Yinhang Road, near 'XinJiangWanCheng' Station), previously named by Blinq Hotel.
- **Registered regular participants** will be housed at the SCMS guest house (on Campus, the gate is on Xuede Road and close to 'XinJiangWanCheng' station).

Figure 3: New University Cafeteria

Figure 4: Ufun Shopping Mall

Figure 5: SCMS Guest House

Figure 6: Fraser Place

Schedules

Conference

	24 Jun (Mon)	25 Jun (Tue)	26 Jun (Wed)	27 Jun (Thu)	28 Jun (Fri)
09:00-9:30	Opening				
09:30-10:30	Uhlmann	Lassas	Salo	Yamamoto	Li
10:30-11:00	*Tea*	*Tea*	*Tea*	*Tea*	*Tea*
11:00-12:00	Qian	Jin	Monard	Triki	Harrach
14:00-14:50	Zhang	Zhou		Liu	
14:50-15:20	*Tea*	*Tea*		*Tea*	
15:20-16:10	Yang	Wang		Saksala	
16:10-17:00				Lu	

School

	29 Jun (Sat)	30 Jun (Sun)	1 Jul (Mon)	2 Jul (Tue)	3 Jul (Wed)
09:00-10:00	Harrach	Triki	Yang	Oksanen	Qian
10:00-10:30	*Tea*	*Tea*	*Tea*	*Tea*	*Tea*
10:30-11:30	Oksanen	Yang	Triki	Qian	Harrach
14:00-15:00	Yang	Oksanen	Harrach	Triki	Qian
15:00-15:30	*Tea*	*Tea*	*Tea*	*Tea*	*Tea*
15:30-16:30	Triki	Harrach	Oksanen	Qian	Yang

Talks, Courses, and Posters

Conference Talks

- **Bastian Harrach** (Frankfurt) *Monotonicity and Convexity in inverse coefficient problems*

  Several applications in medical imaging and non-destructive material testing lead to inverse elliptic coefficient problems, where an unknown coefficient function in an elliptic PDE is to be determined from partial knowledge of its solutions. This is usually a highly non-linear ill posed inverse problem, for which unique reconstructability results, stability and resolution estimates and global convergence of numerical methods are very hard to achieve. In this talk we will review some recent results on Loewner Monotonicity and Convexity that may help in overcoming these issues.

- **Bangti Jin** (CUHK) *Solving Nonlinear Inverse Problems Using Neural Networks and Conditional Stability*

  Using neural networks to solve nonlinear inverse problems has become very popular in recent years. However, often the reconstruction algorithms do not have theoretical justifications. Inspired by conditional stability estimates, we shall propose novel reconstruction algorithms for several PDE inverse problems and discuss their convergence behavior. We illustrate the idea on conductivity imaging and point source identification.

- **Peijun Li** (CAS) *Stability for inverse random source problems*

  Abstract: In the field of inverse problems, the estimation of an unknown source term from indirect observations is a fundamental challenge. Random sources add another level of complexity to this problem due to their uncertainties. In this talk, we will focus on the stability estimates for inverse random source problems of wave equations. An overview will be provided on the existing results for

estimating the stability of the solution in deterministic settings, and our recent findings will be presented for the stochastic case.

- **Matti Lassas** (Helsinki) *Geometric methods for manifold learning*

  We consider the invariant manifold learning (that is, the geometric Whitney problem) on how a Riemannian manifold can be constructed to approximate a given discrete metric space. This problem is closely related to invariant manifold learning, where a Riemannian manifold (M, g) needs to be approximately constructed from the noisy distances $d(X_j, X_k)+\eta_{jk}$ of points $X_1, X_2, \ldots, X_N$, sampled from the manifold M. Here, $d(X_j, X_k)$ are the distance of the points $X_j, X_k \in M$ and η_{jk} are either deterministic or random measurement errors. To study this problem we consider also learning of submanifolds of the high dimensional Euclidean spaces. We also consider applications of the results in inverse problems encountered in medical and seismic imaging. In these problems, an unknown wave speed in a domain needs to be determined from indirect measurements. In geometric terms, this corresponds to the reconstruction of the Riemannian metric associated with the wave velocity from the wave kernel (or the heat kernel) measured in a subset of the domain. The presented results have been done in collaboration with C. Fefferman, S. Ivanov, Y. Kurylev, J. Lu, and H. Narayanan.

- **Yikan Liu** (Kyoto) *Revisit and recent progress of inverse problems for subdiffusion equations*

  The last decade has witnessed explosive developments and gradual saturation of nonlocal PDEs with fractional derivatives in time. Along with the establishment of linear theory especially for subdiffusion equations, there have been extensive studies on related inverse problems, and abundant achievements have been made from both theoretical and numerical aspects.

  Starting with a brief survey on known properties of subdiffusion equations, we revisit their applications to the corresponding inverse problems (backward problems, inverse source problems, etc.), keeping an eye on their similarity and difference with non-fractional counterparts. Next, we share several latest results in this field, including

  1. recovery of multiple parameters in multi-term subdiffusion equations by short-time solution behavior,
  2. simultaneous determination of orders in coupled systems of subdiffusion equations by observing a single component, and

3. backward problem for subdiffusion systems.

- **Jinpeng Lu** (Helsinki) *Quantitative stability of inverse spectral problems*

  The inverse spectral problem concerns the determination of a Riemannian manifold from the spectral data $(\lambda_j, \phi_j|_B)$ of its Laplacian measured on B where B is either the boundary or an open subset of the manifold. The existing methods for the unique solvability of the problem in the general case consist of two steps: the spectral data determine the local distance function representation on B via the unique continuation for the wave operator, and the distance representation on B determines the metric structure of the manifold. We discuss recent progress on the stability of these constructions and consequently on the quantitative stability of the inverse spectral problem. With these improved stability results, we consider the inverse spectral problem for Riemannian orbifolds, that is, spaces which are locally isometric to the quotient of Riemannian manifolds by finite group action. This talk is based on joint works with D. Burago, C. Fefferman, S. Ivanov, M. Lassas, H. Narayanan and T. Yamaguchi.

- **Francois Monard** (UCSC) *X-ray transforms, transport twistor spaces and their blowdown maps*

  Some problems of integral geometry are concerned with the recovery of functions and tensor fields from their integrals along families of curves on a manifold. On Riemannian surfaces, where the curves considered are geodesics, the resolution of these problems has passed through important notions (so-called "fiberwise holomorphicity") which were hinting at an underlying complex-geometric picture, although the latter was only clarified in recent work of Jan Bohr and Gabriel Paternain.

  Given a Riemannian surface (M, g) with unit tangent bundle SM, Bohr-Paternain construct an associated "transport twistor" space, a complex surface containing SM as a boundary component, and where the complex structure degenerates along the geodesic flow. Such a construction has allowed to build correspondences between "fiberwise holomorphic" geodesically invariant distributions on SM, and genuine holomorphic functions on twistor space. This viewpoint has already served as a helpful conceptual tool to reframe and approach geometric inverse problems questions, and leaves many open questions regarding the properties, classification, and rigidity of transport twistor spaces, and their further use in inverse problems.

In this talk, I will report on recent work (joint with Paternain and Bohr, based on arXiv:2403.05985) showing that the degeneracy mentioned above can be "resolved" via maps into $\mathbb{C}^2$, at the price of having these maps have blowdown-type singularities (i.e., akin to maps associated with blowups), in a sense to be clarified in the talk. I will discuss the construction of these maps in some situations, along with some of their uses.

- **Jianliang Qian** (MSU) *Hadamard integrators for wave equations in inhomogeneous media at high frequencies*

  We first present a butterfly-compressed representation of the Hadamard-Babich (HB) ansatz for the Green's function of the high-frequency Helmholtz equation in smooth inhomogeneous media. For a computational domain discretized with N discretization cells, the proposed algorithm first solves and tabulates the phase and HB coefficients via eikonal and transport equations with observation points and point sources located at the Chebyshev nodes using a set of much coarser computation grids, and then butterfly compresses the resulting HB interactions from all N cell centers to each other. The overall CPU time and memory requirement scale as $O(N \log^2 N)$ for any bounded 2D domains with arbitrary excitation sources. A direct extension of this scheme to bounded 3D domains yields an $O(N^{4/3})$ CPU complexity, which can be further reduced to quasi-linear complexities with proposed remedies. The scheme can also efficiently handle scattering problems involving inclusions in inhomogeneous media. Although the current construction of our HB integrator for the Helmholtz equation does not accommodate caustics, the resulting HB integrator itself can be applied to certain sources, such as concave-shaped sources, to produce caustic effects. Compared to finite-difference frequency-domain (FDFD) methods, the proposed HB integrator is free of numerical dispersion and requires fewer discretization points per wavelength. As a result, it can solve wave-propagation problems well beyond the capability of existing solvers. Furthermore, we then present Hadamard integrators for time-dependent wave equations and develop the Huygens secondary-source principle based propagator to treat time-dependent wave equations with caustics. The talk is based on joint works with Dr. Yang Liu, Jian Song, Robert Burridge, Jin Cheng and Yuxiao Wei.

- **Teemu Saksala** (NCSU) *Hyperbolic inverse problems with time dependent and time independent coefficients*

  In this talk we will discuss the differences in the methodology of determin-

ing time-dependent and time-independent coefficients appearing in a hyperbolic equation in a Riemannian manifold. The talk is based on two recent research projects: 1) We will prove that a local source-to-solution map of a hyperbolic partial differential operator on a complete Riemannian manifold (no boundary, and possible non-compact) determines a) the topology and the geometry of the manifold uniquely and b) the lower order time-independent coefficients up to a natural obstruction. 2) We will prove that under certain geometric assumptions the knowledge of a partial Cauchy data set uniquely determines time-dependent lower order coefficients appearing in a hyperbolic initial / boundary value problem. This talk is based on joint works with Boya Liu (NC State University), Andrew Shedlock (NC State University) and Lili Yan (University of Minnesota).

- **Mikko Salo** (Jyväskylä) *Formally determined inverse problems for the wave equation*

  We discuss a method for solving inverse boundary problems for various wave equations in formally determined settings. This is joint work with Lauri Oksanen and Rakesh.

- **Faouzi Triki** (Grenoble) *An improved spectral inequality for sums of eigenfunctions*

  In the talk we revisit the problem of unique continuation of a finite sum of eigenfunctions of an elliptic operator in a divergence form from an open set to the whole domain. We show that the problem is of hyperbolic type for a single eigenfunction, and of elliptic type for sums that have two and more eigenfunctions. This inverse problem can be found in many engineering applications and is also related to control theory for the heat equation.

- **Gunther Uhlmann** (Washington) *The Fractional Anisotropic Calderon Problem*

  We survey some results on this problem.

- **Yiran Wang** (Emory) *Scattering Rigidity Results for the Minkowski Spacetime*

  We are interested in the possibility of recovering spacetime structures from the observation of light signals. The problem is partly motivated by applications in cosmology and general relativity. In this talk, we consider small metric perturbations of the Minkowski spacetime, and discuss what can be recovered from the scattering relation of null geodesics between two Cauchy surfaces. On the linearization level, the problem is related to an X-ray transform in Lorentzian

geometry, called the light ray transform. We discuss recent developments and some challenges in solving the nonlinear problem.

- **Masahiro Yamamoto** (Tokyo) *Inverse problems for mean field game equations*

  In a bounded domain $\Omega \subset \mathbb{R}^d$, $d \geq 1$, over a time interval $(0, T)$, we consider mean field game equations whose principal coefficients depend on the time and state variables with a general Hamiltonian. We discuss several types of inverse problems.

  We first prove the Lipschitz stability in $\Omega \times (\epsilon, T - \epsilon)$ with given $\epsilon > 0$ for the determination of the solutions by the associated Dirichlet data on an arbitrarily chosen subboundary of $\partial\Omega$. Next we prove the Lipschitz stability for an inverse problem of determining spatially varying factors of source terms and a coefficient by extra boundary data and spatial data at an intermediate time.

  This is based on joint works with Professor Oleg Imanuvilov (Colorado State University) and Professor Hongyu Liu (City University of Hong Kong).

References

1. H. Liu, O.Y. Imanuvilov and M. Yamamoto, *Unique continuation for a mean field game system,* Appl. Math. Letters, Volume 145, November 2023, 108757.
2. H. Liu and M. Yamamoto, *Stability in determination of states for the mean field game equations,* Comm. Anal. Comp., **1** (2023), no. 2, 157–167.
3. M. Yamamoto, *Carleman estimates for parabolic equations and applications,* Inverse Problems **25** (2009) 123013

- **Yang Yang** (MSU) *Computational Boundary Control Methods for Acoustic Inverse Boundary Value Problems*

  We consider the acoustic inverse boundary value problem of recovering a sound speed from near field data represented by the Neumann-to-Dirichlet map. We develop linearized boundary control methods to analyze the stability and reconstruction. The analysis leads to reconstructive algorithms that are numerically validated. This is joint work with Lauri Oksanen (University of Helsinki) and Tianyu Yang (Michigan State University).

- **Hai Zhang** (HKUST) *A theory of computational resolution limit for parameter estimation problems*

It is well-known that the resolution of optical imaging system is fundamentally limited by the optical wavelength. Based on this, Rayleigh proposed the Rayleigh criterion on the minimum resolvable distance between two point sources, the so called Rayleigh limit. Although widely used in the practice, this limit is not so useful for images that are subject to elaborated data processing. To remedy this, we develop a theory of computational resolution limit to characterize the fundamental resolution limit from the approximation theory point of view. The theory can be used to explain the phase transition phenomenon in the reconstruction problem. The theory can be extended to the parameter estimation problem for Gaussian mixture models. We demonstrate that with limited number of samples, there is a fundamental limit to the resolvability of the number of Gaussian components in the mixture. This limit depends crucially on the separation distance of the Gaussian means and is termed as the computational resolution limit for the model order estimation problem. Its consequence in the phase transition phenomenon in the estimation problem is also demonstrated through numerical experiments.

- **Hanming Zhou** (UCSB) *Stability and statistical inversion of travel time tomography*

  In this talk, we consider the travel time tomography problem for conformal metrics on a bounded domain, which seeks to determine the conformal factor of the metric from the lengths of geodesics joining boundary points. We establish forward and inverse stability estimates for simple conformal metrics under some a priori conditions. We then apply the stability estimates to show the consistency of a Bayesian statistical inversion technique for travel time tomography with discrete, noisy measurements. The talk is based on joint work with Ashwin Tarikere.

Mini courses

- **Bastian Harrach** (Frankfurt) *Inverse problems for elliptic partial differential equations*

  1. Introduction to inverse problems for elliptic PDEs
  2. Finite Element Methods for inverse problems in elliptic PDEs
  3. The Monotonicity Method for inclusion detection
  4. Uniqueness results for finitely many measurements and convex reformulations

- **Lauri Oksanen** (Helsinki) *Introduction to the Boundary Control method*

  The Boundary Control method can be used to solve several coefficient determination for wave equations. Furthermore, similar inverse problems for many other partial differential equations can be reduced to these problems. This is the case for inverse problems for heat and non-stationary Schrödinger equations, as well as for several fractional equations. Inverse problems for linear elliptic equations on a wave guide, and some non-linear elliptic equations are also covered by this theory. These lectures will give an introductory exposition of the method.

- **Jianliang Qian** (MSU) *Fast Algorithms for geophysical inverse problems*

  1. Overview of fast algorithms for geophysical inversion.
  2. Eikonal solvers and traveltime tomography
  3. Gaussian beams for high frequency waves
  4. Level-set methods for potential-field inversion

- **Faouzi Triki** (Grenoble) *Inverse source problems*

  The lecture provides some recent mathematical results and materials on inverse source problems in scattering theory. In general, uniqueness can not be guaranteed at a fixed energy. Therefore, additional information is required for the source in order to obtain a unique solution for the inverse problem. The first part of the lecture focuses on the problem of recovering point sources from a single boundary measurement. In the second part multifrequency measurements are used to determine general compactly supported sources.

- **Yang Yang** (MSU) *Introduction to Regularization on Reproducing Kernel Hilbert Spaces*

  Regularization techniques play a crucial role in inverse problems by mitigating the ill-posedness and in machine learning by preventing overfitting. This lecture series will give a brief introduction to regularization techniques within the framework of Reproducing Kernel Hilbert Space (RKHS). We will explore the theoretical foundations of regularization methods in RKHS as well as applications to inverse problems and machine learning.

Posters

- **Martin Sæbye Carøe** (DTU) *Stability estimates for the truncated Fourier transform*
- **Xinyue Luo** (Fudan) *Precision-preserving Compression of Scientific Data: Learn Mechanism from Data*
- **Johannes Wagner** (Frankfurt) *Convex parametrization of lung image data via variational autoencoders – A step towards global minimization*

Practical Information

Transportation

The Metro system in Shanghai is highly developed and covers most of the urban regions. It is punctual but might get crowded during rush hours. On the other hand, the journey from Pudong airport to the city is very time consuming.

Taxi is more expensive and faster. But sometimes it is hard to find one. Please have the address handy and show it to the driver.

In addition, there is a super fast maglev line from Pudong airport to 'LongYang Road' station of Metro Line 2 (in the eastern part of the city). Please remember to have your boarding pass or your itinerary with you for the concession price.

The airport shuttle bus for Pudong Airport is cheaper than the cab and faster than the Metro. But it might be tricky for non-Chinese speakers.

Fudan Universtiy consists of 4 campuses. The conference will take place in **Jiangwan Campus** where the Shanghai Centre for Mathematical Sciences (SCMS) is located.

Routes

- From Pudong Airport to Jiangwan Campus / Fraser Place/ SCMS Guest House:
  - By Metro: Take Line 2 at the airport, transfer to Line 10 (northbound) at 'Nanjing East Road', take off at 'XinJiangWanCheng' for Fraser Place and SCMS Guest House or at 'Guofan Road' for the SCMS building.
  - By Taxi: It costs approximately 200 yuan from Pudong Airport to Jiangwan Campus / Fraser Place / SCMS Guest House.
  - By Maglev: Take the maglev line to 'Longyang Road'. Continue to take a taxi or Metro Line 2 afterwards.

  - Shuttle Bus: Take Shuttle Bus Line 4 and take off at 'WuJiaoChang', continue to take Metro Line 10 (northbound).

- From Hongqiao Airport / Rail Station to Jiangwan Campus / Fraser Place / SCMS Guest House :

  - By Metro: Take Line 10 at the airport / rail station and take off at 'XinJiangWanCheng' for Fraser Placeand SCMS Guest House or at 'Guofan Road' for the SCMS building.
  - By Taxi: It costs 100-150 yuan from Hongqiao Airport / Rail Station to Jiangwan Campus / Fraser Place / SCMS Guest House.

Dining Options

- The old university cafeteria is open to visitors. But a 50% surcharge applies.
- The Ufun shopping mall, between Fraser Place and Jiangwan Campus, offers a decent variety of restaurants and supermarkets.
- There are many cheaper options for a quick meal around the west gate of the campus.
- Jiangwan Campus is close to WuJiaoChang (only a few stops by Metro Line 10), one of the leading sub-centres of Shanghai with numerous restaurants, bars, and shopping malls.

Minor Remarks

- Do NOT expect the taxi drivers speak English. It is important to have with you the address or name of your destination in Chinese.
- A valid passport or national ID card is always required for check-in in a hotel.
- Swimming and fishing in the lakes on campus is prohibited!

Emergency Contacts

- Police: 110
- Ambulance: 999