SCMS Seminar

SYMMETRY VERSUS COLLAPSING UNDER BOUNDED CURVATURE

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Lecture

Time: 15:00-16:00, Thursday, May 09, 2019

Venue: Room 106, Shanghai Center for Mathematical Sciences

Abstract: A very beautiful aspect of collapsing phenomena under bounded sectional curvature, founded by Cheeger-Gromov and Cheeger-Gromov-Fukaya, is "Collapsing implies symmetry, and vice versa! ".

Roughly speaking, if there is a manifold M with sectional curvature |K_M|<= 1, then "1-ball B(p,1) has volume <<1 for any p in M" implies there is a torus action on B(p,1) that is isometric in a nearby metric g_\epsilon. If there is a such action, then a family of metrics under bounded sectional curvature can be constructed, such that M collapses to a lower dimensional space.
The terminology describing such phenomena is called the flat structure and nilpotent Killing structure, developed by Cheeger-Gromov and Cheeger-Groov-Fukaya respectively. Recently, such structure has draw much attention from people in study of collapsing of K3 surfaces and G2 manifolds.

We prove that on one fixed smooth manifold, if two metrics with bounded sectional curvature are bi-Lipchitz equivalent and sufficiently collapsed, then up to a diffeomorphism, the underlying nilpotent Killing structures (including flat structures) coincide with each other.

As for a special case, if there are two torus actions whose orbits realize all collapsing directions of two metrics g_1 and g_2, where 1/L_0 g_1<=g_2<=L_0 g_1, then the two actions must be conjugate or one fits another. Note that we do not require the C^1-closeness of them.

Our result improves Cheeger-Fukaya-Gromov's local compatibility result and has applications on collapsed manifolds with (lower) bounded Ricci curvature. The above result has been published online on International Mathematics Research Notices. https://doi.org/10.1093/imrn/rnz023.