

MARTIN BOUNDARY COVERS FLOYD BOUNDARY

Fudan Topology Seminar

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Time: Thur, Oct. 27, 2022 15:00 - 17:00

Meeting Zoom ID: 853 0188 1524 Password: Fudan2022

Abstract: Consider a transient random walk on a countable group G . The Green distance between two points in the group is defined to be minus the boundary of the probability that a random path starting at the first point ever reaches the second.

The Martin compactification of the random walk is a topological space defined to be the horofunction boundary of the Green distance. It is a topological model for the Poisson boundary.

The Martin boundary typically heavily depends on the random walk; it is thus exciting when for some large class of random walks, the Martin boundary is equivariantly homeomorphic to some well known geometric boundary of the group. Ancona showed in 1988 that this is the case for finitely supported random walks on hyperbolic groups: the Martin boundary is identified with the Gromov boundary.

We generalize Ancona's results to relatively hyperbolic groups: the Martin boundary equivariantly continuously surjects onto the Gromov boundary of any hyperbolic space on which the group acts geometrically finitely (called the Bowditch boundary), and the preimage of any conical limit point is a singleton. When the parabolic subgroups are virtually abelian (e.g. for Kleinian groups) we show that the preimage of a parabolic fixed point is a sphere of appropriate dimension, so the Martin boundary can be identified with a Sierpinski carpet.

A major technical tool is a generalization of a deviation inequality due to Ancona saying the Green distance is nearly additive along word geodesics, which has various other applications, including to comparing harmonic and Patterson-Sullivan measures for negatively curved manifolds and to local limit theorems for random walks.

We do all this using an intermediate construction called the Floyd metric obtained by suitably rescaling the Cayley graph and considering the associated completion called the Floyd compactification. We show that for any finitely supported random walk on a finitely generated group, the Martin boundary surjects to the Floyd boundary, which in turn by work of Gerasimov covers the Bowditch boundary of relatively hyperbolic groups. This is based on several joint works with subsets of Dussaule, Gerasimov, Potyagailo, and Yang.