

2022 SCMS Algebraic Geometry Summer School

Shanghai Center for Mathematical Sciences

Aug. 8-Aug.19, 2022

Speaker: Junyan Cao (University of Nice)

Title: Curvature of direct images and some applications

Abstract: Given a fibration between two projective manifolds, an important tool to study the fibration is the positivity of the (higher) direct image of its relative canonical bundle. I will explain some analytic aspects of it, and also some applications in algebraic geometry.

Content:

1. Notions of positivity of vector bundles, singular hermitian metrics.
2. Multiplier ideal sheaves and Ohsawa-Takegoshi extension theorem.
3. Positivity of direct images of relative pluri-canonical bundle
4. Curvature of higher direct images and Viehweg-Zuo sheaf
5. Some applications: hyperbolicity of canonically polarized or Calabi-Yau family

Reference:

1. J.-P. Demailly: Analytic methods in algebraic geometry
2. M. Păun: Singular Hermitian metrics and positivity of direct images of pluricanonical bundles; arXiv:1606.00174
3. A. Höring: Positivity of direct image sheaves - a geometric point of view
4. C. Hacon, M. Popa, C. Schnell: Algebraic fiber spaces over abelian varieties: around a recent theorem by Cao and Paun; arXiv:1611.08768
5. B. Berndtsson, M. Paun, X. Wang: Algebraic fiber spaces and curvature of higher direct images

Speaker: Damian Brotbek (University of Lorraine)

Title: Hyperbolicity and variation of Hodge structures

Abstract: The goal of this series of lectures is to give an introduction to the theory of hyperbolic manifolds in the sense of Kobayashi. First will present the classical theory and describe the main conjectures of this field. We will then introduce jet spaces and explain how these can be used to prove that general hypersurfaces of high degree are hyperbolic.

Content:

1. Hyperbolic manifolds
2. Jet differentials
3. Hyperbolicity of hypersurfaces

Reference:

1. Jean-Pierre Demailly, Hyperbolic algebraic varieties and holomorphic differential equations.
2. Shoshichi Kobayashi, Hyperbolic complex spaces.
3. Junjiro Noguchi and Jörg Winkelmann, Nevanlinna theory in several complex variables and Diophantine approximation.

Speaker: Andreas Höring (University of Nice)

Title: Positivity of the cotangent bundle

Abstract: While the canonical bundle is at the center of the minimal model program and arguably the most important object in complex algebraic geometry, the cotangent bundle is much more mysterious. Initiated by progress in foliation theory the positivity of the (co)tangent bundle has been more thoroughly investigated in the last years. In these talks I will start by discussing stability of coherent sheaves and the various notions of positivity for vector

bundles. I will then show how to put these notions together to get a numerical criterion for pseudoeffectivity in the case where the first Chern class vanishes. In the last part I will discuss with manifold with pseudoeffective (co)tangent bundles.

Content:

1. Stability of vector bundles on curves
2. Stability of coherent sheaves
3. Positivity of vector bundles
4. Stable vector bundles with $c_1=0$
5. Manifolds with pseudoeffective (co)tangent bundle

Reference:

1. R. Lazarsfeld, Positivity in algebraic geometry
2. Y. Miyaoka, T. Peternell, Geometry of higher-dimensional algebraic varieties
3. S. Druel, A decomposition theorem for singular spaces with trivial canonical class of dimension at most five
4. A. Höring, T. Peternell, Algebraic integrability of foliations with numerically trivial canonical bundle

Speaker: Chen Jiang (SCMS)

Title: Linear systems of line bundles on K3 surfaces and hyperkahler manifolds

Abstract: I will discuss the behavior of linear systems of line bundles (e.g. non-vanishing, base-point freeness, very ampleness) on K3 surfaces and hyperkahler manifolds. I will explain general methods dealing with linear systems such as Reider's theorem, Riemann-Roch theorem, and vanishing theorem, and also methods depending on special geometry properties of K3 and hyperkahler manifolds.

Content:

1. Linear systems
2. Reider's theorem
3. Base-point freeness on K3 surfaces
4. Basics on hyperkahler manifolds
5. Positivity of Riemann-Roch of hyperkahler manifolds
6. Base divisor of line bundles on hyperkahler manifolds

Reference:

1. D. Huybrechts, Lectures on K3 surfaces
2. B. Saint-Donat. Projective models of K3 surface
3. C. Jiang, Positivity of Riemann-Roch polynomials and Todd classes of hyperkahler manifolds
4. U. Riess, Base divisors of big and nef line bundles on irreducible symplectic varieties

Speaker: Osamu Fujino (Kyoto University)

Title: Introduction to the Iitaka conjecture

Abstract: The program for birational classification of higher-dimensional complex algebraic varieties initiated by Shigeru Iitaka around 1970 is called the Iitaka program. I will explain it and discuss the Iitaka conjecture, which claims the subadditivity of the Kodaira dimension for algebraic fiber spaces. I will prove some special cases of the Iitaka conjecture.

Content:

1. Kodaira dimension
2. Iitaka conjecture
3. Iitaka fibrations
4. Vanishing theorems
5. Semipositivity

Reference:

1. O. Fujino, Iitaka conjecture—an introduction
2. K. Ueno, Classification theory of algebraic varieties and compact complex spaces
3. S. Iitaka, Birational geometry for open varieties
4. S. Iitaka, Algebraic geometry. An introduction to birational geometry of algebraic varieties

Speaker: Wenhao Ou (CAS)**Title:** Minimal Model Program for Kaehler 3-folds**Abstract:** We will introduce MMP for Kaehler 3-folds, which was established by Hoering-Peternell.**Content:**

1. Kaehler varieties
2. Criterion for nefness
3. Zariski decomposition theorem
4. Cone theorem
5. Contraction theorem
6. Existence of Minimal Model
7. Existence of Mori fiber space

Reference:

1. Hoering, Peternell. Minimal models for Kähler threefolds
2. Hoering, Peternell. Mori fibre spaces for Kähler threefolds
3. Kollar, Mori. Birational geometry of algebraic varieties

Speaker: Jian Xiao (Tsinghua University)**Title:** Positivity in algebraic and analytic geometry: basics**Abstract:** When working over the field of complex numbers, there is a motto "algebraic is analytic". In this short course, we will see some aspects of this principle by introducing basics on the positivity in algebraic geometry and analytic geometry. The course is planned to include the following topics: current, plurisubharmonic function, Lelong number, positive line bundle, singular metric, multiplier ideal sheaves, Nadel vanishing theorem, etc.

Content:

1. Currents
2. Plurisubharmonic functions and Lelong numbers
3. Positive line bundles
4. Multiplier ideal sheaves
5. Vanishing theorems

Reference:

1. J.P. Demailly: Analytic methods in algebraic geometry
2. R. Lazarsfeld: Positivity in algebraic geometry

Speaker: Songyan Xie (CAS)

Title: Fundamental vanishing theorem of entire curves and applications

Abstract: Key ideas behind the fundamental vanishing theorem of entire curves can be traced back to a seminal paper of André Bloch about one hundred years ago. We will review some historical developments, and applications in complex hyperbolicity problems. Some related positivity results in algebraic geometry will also be discussed.

Reference:

1. Bloch, André Sur les systèmes de fonctions uniformes satisfaisant à l'équation d'une variété algébrique dont l'irrégularité dépasse la dimension
2. Siu, Yum-Tong Hyperbolicity of generic high-degree hypersurfaces in complex projective space.
3. Yamanoi, Katsutoshi Kobayashi hyperbolicity and higher-dimensional Nevanlinna theory.