TOPICS ON NEVANLINNA THEORY AND COMPLEX HYPERBOLICITIES

Conference

(July 25 - July 27, 2019)

1. Schedule

	July 25	July 26	July 27
9:00-9:50	Yum-Tong Siu	Nessim Sibony	Junjiro Noguchi
9:50-10:10	Coffee break		
10:10-11:00	Ngaiming Mok	Sai-Kee Yeung	Shiferaw Berhanu
11:00-11:10	Coffee break		
11:10-12:00	Tuen-Wai Ng	Ya Deng	Erwan Rousseau
	Lunch Break		
2:00-2:50	Steven Lu	Shengli Tan	
2:50-3:00	Coffee break		
3:00-3:50	Gordon Heier	Julie Wang	
3:50-4:30	Kenneth Archer	Coffee break	
4:30-5:20	Amos Turchet	Jianhua Zheng	

2. Title & Abstract

Kenneth Ascher

Title: Hyperbolicity of varieties of log general type

This talk will survey some recent results regarding hyperbolicity of varieties of log general type. There are several classical results showing that positivity of the cotangent bundle implies various notions of hyperbolicity for projective varieties coming from algebraic, arithmetic, and differential geometry. The goal of this talk is to review these results and discuss recent work which generalizes these results for log pairs / quasi-projective varieties. Time permitting we will also discuss the function field case. This is joint work with K. DeVleming and A. Turchet.

Shiferaw Berhanu

Title: On unique continuation for CR mappings and solutions of some elliptic equations

We will discuss some old and recent results on the unique continuation problem for CR mappings between CR manifolds and the related problem of boundary unique continuation for solutions of a class of elliptic partial differential equations.

Ya Deng

Title: Hyperbolicity of moduli spaces of higher dimensional complex manifolds

For maximal variational smooth families of polarized manifolds with semi-ample canonical bundle, the Viehweg-Zuo hyperbolicity conjecture says that the base spaces of such families should be of log general type. This deep conjecture was proved by Campana-Paun in 2015, building on the previous fundamental work by Viehweg-Zuo in 2002. In this talk I will show that those base spaces are furthermore pseudo Kobayashi hyperbolic, i.e. Kobayashi hyperbolic modulo a Zariski closed subset, as predicted by the famous Lang conjecture. This in particular proves a conjecture by Viehweg-Zuo in 2003: moduli spaces of polarized manifolds with semi-ample canonical bundle are Brody hyperbolic. I will also present another related work (jointly with Dan Abramovich) on the Kobayashi hyperbolicity for moduli spaces of minimal general type manifolds, which generalizes the previous celebrated work by To-Yeung in 2015 on the Kobayashi hyperbolicity for moduli of canonically polarized manifolds.

Gordon Heier

Title: On a generalized Schmidt subspace theorem and degeneracy of integral points

In this talk, we will present a generalized version of Schmidt's subspace theorem for closed subschemes in general position in terms of Seshadri constants with respect to a fixed ample divisor. Our proof builds on previous work by Evertse and Ferretti, Corvaja and Zannier, and others, and uses techniques from algebraic geometry. As an application, we will present a degeneracy theorem for integral points on the complement of a union of nef effective divisors. Its proof hinges on applying our Schmidt-type theorem with a well-situated ample divisor realizing a certain lexicographical minimax. This is joint work with Aaron Levin.

Steven Lu

Title: $\mathbb{C}^n$-dominability of hyperkaehler manifolds

Complex projective or compact kaehler manifolds with trivial canonical class are up to a finite unramified cover, products of simple hyperkaehler manifolds, Calabi-Yau manifolds and compact complex tori, the former two types being simply connected. Not much is known concerning the lack of hyperbolicity of these simply connected manifolds besides the case of K3-surfaces. One expectation is that entire holomophic curves can be Zariski dense in any such variety, i.e. fails to be algebraically degenerate. The only known method for showing this for such a variety is to construct dominating meromorphic or holomorphic maps from $\mathbb{C}^n$ and so far this has only been done for certain K3 surfaces. In this talk, I will show how to construct such dominating maps for certain well-known classes of hyperkaehler manifolds, generalizing the results for K3 surfaces. This is joint work with Ljudmila Kamenova.

Ngaiming Mok

Title: Uniformization of Subvarieties of Finite-Volume Quotient Spaces of Bounded Symmetric Domains

The Siegel upper half plane Hg belongs, up to biholomorphic equivalence, to the set of bounded symmetric domains, on which a great deal of mathematical research is taking place. Especially, finite-volume quotients of bounded symmetric domains, which are naturally quasi-projective varieties, are objects of immense interest to Several Complex Variables, Algebraic Geometry, Arithmetic Geometry and Number Theory, and an important topic is the study of covering spaces of algebraic subsets of such quasi-projective varieties. While a lot has already been achieved in the case of Shimura varieties by means of methods of Diophantine Geometry, Model Theory, Hodge Theory and Complex Differential Geometry, techniques for the general case of not necessarily arithmetic quotients = Ω/Γ have just begun to be developed. For

instance, uniformization problems for subvarieties of products of arbitrary compact Riemann surfaces of genus ≥ 2 have hitherto been untractable by existing methods. We will explain a differential-geometric approach leading to various characterization results for totally geodesic subvarieties of finite-volume quotients = Ω/Γ. Especially, we will explain how the study of holomorphic isometric embeddings of the Poincare disk and more generally complex unit balls into bounded symmetric domains can be further developed to derive uniformization theorems for bi-algebraic varieties and more generally for the Zariski closure of images of algebraic sets under the universal covering map.

Patrick, Tuen Wai Ng

Title: Ax-Schanuel type inequalities for functional transcendence via Nevanlinna theory

The Ax-Schanuel Theorem implies that for any Q-linearly independent modulo C entire functions of one complex variable $f_1, \cdots, f_n$, the transcendence degree over C of $f_1, \cdots, f_n; e(f_1), \cdots, e(f_n)$ is at least n+1 where e(z) = $e^{2\pi iz}$ It is natural to ask what happens if one replaces the exponential map e by some other meromorphic functions. In this talk, we will apply Nevanlinna theory to obtain several inequalities of the transcendence degree over C of $f_1, \cdots, f_n; F(f_1), \cdots, F(f_n)$ when f_i's are entire functions with some growth restrictions and F is a transcendental meromorphic function. The results are joint work with Jiaxing Huang.

Junjiro Noguchi

Title: Nevanlinna theory for semi-abelian varieties and some arithmetic properties

We will see a direct relation at the proof level between the Nevanlinna theory (a Big Picard Theorem, '81 by the author) and an arithmetic result on torsion points in semi-abelian varieties (Manin-Mumford Conjecture, M. Raynaud's Theorem, '83). It is interesting to see the role of ``o-minimal structure'' connecting the two subjects. We then discuss the Nevanlinna theory of holomorphic sections of relative semi-abelian varieties defined over curves or a disk of complex plane. A typical and interesting case is Legendre's elliptic curves define over complex plane minus 0 and 1, where will use Yamanoi's S.M.T. We then discuss the higher dimensional case, where we will find some new problems.

Erwan Rousseau

Title: Automorphisms of foliations

We will discuss in various contexts the transverse finiteness of the group of automorphisms/birational transformations preserving a holomorphic foliation. This study also provides interesting consequences for the distribution of entire curves on manifolds equipped with foliations. This is a work in progress with F. Lo Bianco, J.V. Pereira and F. Touzet.

Nessim Sibony

Title: Generic foliations on kähler surfaces

I will discuss some recent results on the distribution of leaves for generic foliations by Riemann surfaces, on compact kähler surfaces.

Generically the leaves are uniformized by the unit disc. The problem is to understand how they distribute in the manifold.

The talk is based on joint works with J.E Fornaess and with T.C Dinh and more recently results with T.C Dinh and V.A Nguyen. The main new tool is the Theory of Densities developed with T.C Dinh.

Yum-Tong Siu

Title: Nevanlinna Theory and Its Application to the Abundance Conjecture

Will start with some discussion of Nevanlinna theory and then focus on the application of Nevanlinna theory and the technique of Gelfond-Schneider-Lang-Bombieri to the analytic approach to the abundance conjecture.

Shengli Tan

Title: Slope of Non-algebraic holomorphic foliations

For a holomorphic foliation on a projective smooth surface, we introduced their Chern numbers which are nonnegative birational invariants. If the foliation is of general type, we can define its slope similar to that of a fibration. We are going to talk about the upper and lower bounds of the slopes for non-algebraic foliations by considering double covers of some simple foliations. This is a joint work with Jie Hong and Jun Lu.

Amos Turchet

Title: Algebraic Hyperbolicity of fibered threefolds

We discuss algebraic hyperbolicity of fibered threefold whose generic fiber is the complement of a reducible divisor in a smooth projective surface. These generalize results of Corvaja and Zannier and of the author. The main theorem deals with ramified covers of a G_m^2 bundle and builds on an explicit bound for the number of multiple zeroes of a polynomial with non-constant coefficients evaluated at S-units. This is joint work with Laura Capuano.

Julie Tzu-Yueh Wang

Title: Greatest common divisors of analytic functions

We discuss upper bounds for the counting function of common zeros of two analytic functions in various contexts. The proofs and results are inspired by recent work involving greatest common divisors in Diophantine approximation, to which we introduce additional techniques to take advantage of the stronger inequalities with ramification terms available in Nevanlinna theory. In particular, we prove a general version of a conjectural ``asymptotic gcd" inequality. We also extend the gcd inequality for tori by Noguchi-Winkelman-Yamanoi to a more general situation. The talk is based on a joint work with Aaron Levin and a joint work with Yu Yasufuku.

Sai-Kee Yeung

Title: Complex hyperbolicity and augmented Weil-Peterson metrics on moduli spaces

We will explain some joint work with Wing-Keung To on complex hyperbolic properties of various moduli spaces of higher dimensional polarized projective algebraic manifolds. The complex hyperbolic properties could be described in terms of holomorphic sectional curvature and Ricci curvature, in terms of Kobayashi hyperbolicity and measure hyperbolicity, or in terms of (log)-general type properties and (log)-Kodaira dimension. We would explain various aspects of the theory from the point of view of Weil-Petersson metric and its generalizations.

Jianhua Zheng

Title: Argument Distribution of Holomorphic Curves

As we know, there are a lot of beautiful results in the value distribution of meromorphic functions, which have been investigated for more than one hundred years. It is nature to consider whether for holomorphic curves those results are still true. In this talk, we consider the existence of singular directions of holomorphic curves. In terms of the potential theory of Erememko and Sodin in the value distribution, we establish the second main theorem for a holomorphic curve in an angular domain and the existence of singular direction follows from this theorem. In fact, in view of the method of limit set of subharmonic functions of Sodin, we also offer another proof of the existence of singular directions. However, from the potential methods we do not seem to consider the case of non-generated holomorphic curves. We introduce the Nevanlinna characteristic of holomorphic curves in an angular domain and from this, we can consider the singular directions of non-generated holomorphic curves. Eremenko and Tu proved independently the existence of Julia directions.