MINIWORKSHOP ON TILING AND SPECTRAL ANALYSIS

Time：November 1st to 3rd

Venue: Room 106, SCMS

Talks and Abstacts

Time: November 2nd 10:00-10:50
Speaker: Mihalis Kolountzakis
Title: Simultaneous tiling and comfortable packing

Abstract: It has recently been proved by Grepstad, Kolountzakis and Spyridakis (2025) that if T and P are lattices in d-dimensional Euclidean space and vol(T) < vol(P) then there exists a bounded, measurable set E (in fact, a finite union of polytopes) such that E tiles when translated by T and packs (non-overlapping translates) when translated by P. It is of importance for applications, mainly to Gabor analysis, that the P-packing is "comfortable": the P-translates of E should not be just disjoint but should also have a positive distance from each other. In this talk I will describe joint work with Andrei Caragea and Goetz Pfander in which we determine exactly when a pair of lattices T and P, with vol(T) < vol(P) have such a set E which tiles with T and packs with P comfortably. The answer is that this is always possible when the sum T+P is not a lattice, while in the commensurable case, when H=T+P is a lattice, this is possible exactly when [H:P] is at least [H:T]+d.

Time: November 2nd 11:00-11:50

Speaker: Junjie Zhu

Title: Spectrality and curvature of planar curves

Abstract: A finite Borel measure is called spectral if its associated L^2 space admits an orthonormal basis of complex exponentials. Many familiar examples of spectral measures supported on curves arise from straight line segments, raising the question of whether curvature can obstruct spectrality. In this talk, we investigate this question for normalized arc-length measures on planar curves. We discuss a proof that a regular (C^2) planar arc can be spectral only if it is a straight-line segment, highlighting the key geometric argument.

Time: November 2nd 15:00-15:50

Speaker: Kailing Lai

Title: Common tiling functions with small support

Abstract: For N lattices in $\R^d$ with volume 1 and pairwise trivial intersections, every nonzero common tiling function has support diameter $\Omega(N^{1/d})$, while for lattice families whose fundamental domains have uniformly bounded diameters, the standard convolution construction gives an $O(N)$ upper bound, leaving a gap that has remained open since the work of Kolountzakis and Wolff (1999). We close this gap by constructing, for every $d\geq 2$ and all $N\ge2$, lattice families satisfying the same volume and intersection conditions that admit a nonnegative common tiling function with support diameter $O(N^{1/d})$, thereby also answering Question 1 of Kolountzakis and Papageorgiou (2022).

We also obtain the optimal $O(\sqrt N)$ upper bound by constructing, for any prescribed family of plane lattices whose volumes lie in a fixed bounded set independent of N, a pairwise trivially intersecting family with the same respective volumes and with bases arbitrarily close to suitable bases of the prescribed lattices.

Time: November 2nd 16:00-16:50

Speaker: Sha Wu

Title: Spectrality of Weighted Measures on Two Line Segments

Abstract: This talk studies the spectrality of measures with positive integrable densities supported on two line segments. We characterize spectrality for different geometric configurations and prove that every such spectral measure admits a spectrum contained in a straight line. We also present examples on unions of more than two segments that admit no line spectrum, together with growth estimates for orthogonal exponential sets associated with arc-length measures on finite unions of rectifiable curves. This talk is based on joint work with Mihalis Kolountzakis and Ruxi Shi.

Host: Ruxi Shi **Contact:** Yiling Liu (liuyiling@fudan.edu.cn)