

# Topological and probabilistic methods in low-dimensional dynamics

Gu Lecture Hall, SCMS

	Monday (10.21)	Tuesday (10.22)	Wednesday (10.23)	Thursday (10.24)	Friday (10.25)
9:30- 10:20	M. Benedicks	R. Sharp	D. Feng	J. An	S. Gan
10:20- 10:50	Tea Break	Tea Break	Tea Break	Tea Break	Tea Break
10:50- 11:40	H. Bruin	K. Baranski	B. Barany	H. Takahasi	Y. Cao
<b>Lunch</b>					
14:00- 14:50	Q. Zhou	L. Shu	J. Xue	M. Todd	W. Wu
15:00- 15:50	Z. Zhang	R. Gao	L. Xu	Y. Son	J. Kotus
15:50- 16:20	Tea Break	Tea Break (Group Photo)	Tea Break	Tea Break	Tea Break
16:20- 17:10	Y. Shi	E. Kin	G. Liao	L. Liao	M. Pollicott
17:20— 18:10		Banquet (Yanyuan Hotel)	J. Chen		

### **Jinpeng An**

Title: Nondense orbits for affine maps and applications.

Abstract: Let  $f$  be an affine map on a homogeneous space  $X$  that is the quotient of a Lie group by a discrete subgroup,  $Z$  be a subset of  $X$ , and consider the set  $E(f,Z)$  of points in  $X$  whose forward orbits under  $f$  do not enter some neighborhood of  $Z$ . For important cases,  $f$  is ergodic with respect to the Haar measure on  $X$ , and hence  $E(f,Z)$  has zero measure. We will discuss a recent result stating that if  $Z$  is a countable union of submanifolds satisfying certain transversality conditions, then  $E(f,Z)$  has full Hausdorff dimension. This generalizes/unifies some previous results of Dani, Kleinbock, etc. We will also discuss the geometric and number-theoretic applications. In particular, the result can be applied to show that for any point  $y$  in a locally symmetric space  $Y$  of noncompact type and any countable subset  $C$  of  $Y$  not containing  $y$ , the set of directions at  $y$  such that the closure of the geodesic through  $y$  in that direction does not meet  $C$  has full Hausdorff dimension in the set of all directions. This is a joint work with Lifan Guan and Dmitry Kleinbock.

### **Krzysztof Baranski**

Title: Singular stationary measures for random interval homeomorphisms

Abstract: We show that the stationary measure for some random systems of two piecewise affine homeomorphisms of the interval is singular, verifying partially a conjecture by Alsedà and Misiurewicz and contributing to a question of Navas on the absolute continuity of stationary measures, considered in the setup of semigroups of piecewise affine circle homeomorphisms. This is a joint work with Adam Śpiewak.

### **Balazs Barany**

Title: Super-exponential condensation without exact overlaps

Abstract: In the dimension theory of self-similar sets and measures, one of the important conjectures is the exact overlap conjecture. The conjecture says if the similarity dimension is strictly larger than the Hausdorff then there is an exact overlap between the functions of the corresponding IFS. The known strongest result in this direction is due to Hochman, who showed that exponential separation suffices for the equality of the Hausdorff and similitude dimensions of self-similar sets and measures. In this talk, we exhibit self-similar sets on the line which are not exponentially separated and do not generate any exact overlaps. This is a joint work with Antti Kaenmäki.

### **Michael Benedicks**

Title: Coexistence phenomena for Hénon maps

Abstract: In the standard Hénon family various coexistence phenomena can occur. In particular there is a positive Lebesgue measure set of parameters such that finitely many attractive periodic orbits and a "strange attractor" coexist. We also get a new approach to the Newhouse phenomenon of infinitely many coexisting attractive periodic orbits.

Also two strange attractors can coexist for maps parameters in the classical Hénon family. This is joint work with Liviana Palmisano.

### **Henk Bruin**

Title: Limit laws for volume-preserving almost Anosov maps and flows.

Abstract: Almost Anosov maps are Anosov maps except for finitely many neutral periodic points, and almost Anosov flows are the flow-equivalent of this. One can use an inducing scheme to regain hyperbolicity. The current operator renewal-type approach to obtain polynomial mixing rates in various dynamical systems requires that the tails of a certain inducing scheme have regular variation. Using detailed analysis of the behaviour near the neutral periodic points, one can compute tail behaviour of these induced maps, and consequently derive stochastic limit laws of the motion. The current methods require particular constraints on the local behaviour near the neutral orbits, but in the volume preserving setting, these are in a sense generic.

### **Yongluo Cao**

Title: The estimates of dimensions for nonconformal repellers.

Abstract: In this talk, we report some recent results about the estimates of dimensions for nonconformal repellers. This is joint work with Pesin and Zhao Yun.

### **Jianyu Chen**

Title: On Coupling Lemma and Stochastic Properties for One-dimensional Expanding Maps

Abstract: In this talk, we establish a coupling lemma for standard families in the setting of piecewise expanding interval maps with countably many branches. This method is particularly powerful for maps whose inverse Jacobian has low regularity and those who does not satisfy the big image property. The main ingredient of our proof is to estimate the average length of standard families under the operations of cutting, iterates and splitting in terms of the characteristic  $\mathcal{Z}$  function. We further conclude the existence of an absolutely continuous invariant measure, the exponential decay of correlations and the almost sure invariance principle (which is a functional version of the central limit theorem). The latter two stochastic properties hold for a large class of unbounded observables, due to our crucial assumption called Chernov's one-step expansion at  $q$ -scale. This is a joint work with Hong-Kun Zhang and Yiwei Zhang.

### **De-Jun Feng**

Title: Dimension of attractors of nonlinear iterated function systems

Abstract: In this talk, we discuss the dimension of the attractors of nonlinear IFS. We prove that the upper box-counting dimension of a  $C^1$  IFS contractor is always bounded above by its singularity dimension. The result extends to all  $C^1$  non-conformal repellers. Furthermore, we consider some nonlinear IFS on the plane such

that the derivatives of all the maps are lower triangular matrices. Under mild assumptions we prove that the Hausdorff and box-counting dimensions of the attractor are equal to its singular dimension for almost all translates of the system. The talk is based on joint work with Karoly Simon.

### **Shaobo Gan**

Title: Lyapunov stable chain recurrence class of flows far from homoclinic tangencies

Abstract: We show that if a  $C^1$  generic vector field  $X$  is far from homoclinic tangencies, then its non-trivial Lyapunov stable chain recurrence class is a homoclinic class. This is a joint work with Jiagang Yang and Rusong Zheng.

### **Rui Gao**

Title: Local stable and unstable sets for positive entropy  $C^1$  diffeomorphisms

Abstract: In this talk, we consider the local stable/unstable sets for any  $C^1$  dynamical system on a compact manifold that admits an ergodic measure with positive entropy. A lower bound of the Hausdorff dimension for the local stable/unstable sets is given in terms of the measure-theoretical entropy and the maximal Lyapunov exponent. This is a joint work with Shilin Feng, Wen Huang and Zeng Lian.

### **Eiko Kin**

Title: Asymptotic translation lengths of pseudo-Anosov maps on the curve complex and fibered 3-manifolds

Abstract: Let  $S$  be an orientable surface. Each mapping class  $f$  in the mapping class group of  $S$  acts by an isometry on the curve complex  $C(S)$ . Then the asymptotic translation length on  $C(S)$  can be defined. It measures the distance between a curve  $c$  and  $f^n(c)$  on average. It is proved by Masur-Minsky that a mapping class  $f$  is pseudo-Anosov if and only if its asymptotic translation length is positive. One can similarly define the asymptotic translation length of  $f$  on the Teichmüller space of  $S$  with the Teichmüller metric, which is equal to the entropy of  $f$ . Let  $M$  be a hyperbolic fibered 3-manifold. We consider sequences of fibers and pseudo-Anosov monodromies for primitive integral classes in the fibered cone of  $M$ . We study if it is possible to obtain a continuous extension of normalized asymptotic translation lengths on the curve complex as a function on the fibered face. An analogous question for normalized entropy has been answered affirmatively by Fried. We show that such an extension in the case of the curve complex does not exist in general by explicit computation for sequences in the fibered cone of the magic manifold. This is joint work with Hyungryul Baik, Hyunshik Shin, Chenxi Wu.

### **Janina Kotus**

**Title:** Lyapunov exponents of transcendental meromorphic function

**Abstract:** Lyapunov exponents of meromorphic functions do not seem to have been studied yet. We are mainly concentrated on the problem for which points in  $J(f)$  the upper Lyapunov exponent  $\chi^+(z) = +\infty$  and the lower Lyapunov exponent  $\chi^-(z) = -\infty$  and how big these sets are. In order to find the points in  $J(f)$  for which  $\chi^+(z) = +\infty$  we start with the escaping set  $I(f) = \{z \in \mathbb{C} : \lim_{n \rightarrow \infty} |f^n(z)| = \infty\}$ . For transcendental meromorphic functions with poles that  $I(f)$  is non-empty and  $J(f) = \partial I(f)$ . Properties of the escaping set of entire or meromorphic functions have been investigated by several authors (e.g. Baranski, Bergweiler, Karpinska, Rippon, Stallard). In particular the Hausdorff dimension of  $I(f)$  has been studied. But it should be pointed out that not for every point  $z \in I(f)$  the upper Lyapunov exponent  $\chi^+(z) = +\infty$ . To estimate the Lebesgue's measure of the set of points in  $J(f)$  with  $\chi^+(z) = +\infty$  we have to concentrate on more specific functions. We consider the family  $f_\lambda(z) := \lambda \tan(z)$ ,  $\lambda \in \mathbb{C} \setminus \{0\}$  and  $z \in \mathbb{C}$ . For different classes of functions in this family, including Misiurewicz maps, we estimate  $\chi^+(z)$  and  $\chi^-(z)$  for almost every  $z \in J(f) = \mathbb{C}$ .

### **Gang Liao**

**Title:** Margulis-Ruelle inequality for general manifolds

**Abstract:** We investigate the Margulis-Ruelle inequality for general Riemannian manifolds (possibly noncompact and with boundary) and show that this inequality always holds under integrable condition.

### **Linmin Liao**

**Title:** Rational dynamical systems on finite extensions of the field of  $p$ -adic numbers

**Abstract:** Let  $K$  be a finite extension of the field of  $p$ -adic numbers. We study the dynamics of a rational map on  $K$ . It is showed that for a rational map of degree at least two without wildly recurrent Julia critical point, the Julia set is the restriction of the Julia set of the same map in the field of complex  $p$ -adic numbers on the projective line of  $K$ . Further, if the rational map is geometrically finite, then the dynamics of the map on its Julia set deleting the grand orbits is topologically conjugate to a countable states Markov shift. As example, we completely describe the dynamics of the map  $f: x \rightarrow 9x(x-1)^2/4$  on the projective line of the field of 2-adic numbers. We underline that such map is the unique cubic postcritically finite polynomial with rational coefficients such that the corresponding Julia set contains a critical point. This is a joint work with Shilei Fan, Hongming Nie and Yuefei Wang.

### **Mark Pollicott**

**Title:** Volume entropy and large circles for flat surfaces

**Abstract:** For a flat surface, or translation surface, we can define the volume entropy in terms of the rate of growth of the circumference of a large circle, whose radial geodesics typically include saddle connection paths. We will discuss asymptotic formulae and the distribution of such circles as the radius tends to infinity.

### **Richard Sharp**

Title: Weighted equidistribution of null-homologous orbits

Abstract: Consider the geodesic flow over a compact manifold with negative sectional curvature. A theorem of Katsuda and Sunada says that the null-homologous periodic orbits become, on average, equidistributed with respect to the measure of maximal entropy as the periods tend to infinity. We show that by introducing a natural weighting we can obtain equidistribution with respect to the volume. We will also discuss extensions to other volume-preserving Anosov flows and more general weightings.

### **Yi Shi**

Title:  $C^r$ -Closing lemma for partially hyperbolic diffeomorphisms with one-dimensional center bundle

Abstract. The  $C^r$ -closing lemma is one well-known problem in the theory of dynamical systems. The problem is to perturb the original dynamical system so as to obtain a  $C^r$ -close system that has a periodic orbit passing through a given point. And this point is called  $C^r$ -closable. In this talk, we prove the  $C^r$ -closing lemma ( $r = 2, 3, \dots, \infty$ ) for partially hyperbolic diffeomorphisms with one-dimensional center bundle: every non-wandering point of these diffeomorphisms is  $C^r$ -closable. Moreover, we will show that  $C^r$ -generic conservative partially hyperbolic diffeomorphisms with one-dimensional center bundle have dense periodic points. This is a joint work with Shaobo Gan.

### **Lin Shu**

Title: A family of diffusions for the stable foliation

Abstract: For a smooth closed negatively curved manifold, we consider a family of diffusions for the stable foliation and show the stationary probability measures on the unit tangent bundle interpolate between the Burger-Roblin measure and the normalized Lebesgue measure. This is a joint work with François Ledrappier.

### **Younghwan Son**

Title: Uniform distribution of generalized polynomials and its applications to sets of recurrence.

Abstract: A generalized polynomial is a real-valued function which is obtained from polynomials by use of bracket operations, sums, and products. The notion of generalized polynomials is the natural extension of conventional polynomials, but generalized polynomials may exhibit quite intricate distributional properties. In this talk, we will present uniform distribution of a large class of generalized polynomials and its applications to sets of recurrence. This is joint work with Vitaly Bergelson and Inger Halland Knutson.

**Hiroki Takahasi**

Title: Existence of large deviations rate function for any S-unimodal map

Abstract: For any unimodal map with negative Schwarzian derivative and with non-flat critical point, we establish the Large Deviation Principle (LDP) for empirical distributions. Our focus is on the infinitely renormalizable case, which was left off in the previous work by Chung, Rivera-Letelier, and the speaker that establishes the LDP for a large class of topologically exact multimodal maps. This is a joint work with Masato Tsujii (Kyushu University).

**Mike Todd**

Title: Escape of entropy

Abstract: In many classical compact settings, entropy is upper semicontinuous, i.e., given a convergent sequence of invariant probability measures, the entropy of the limit measure is at least the limsup of the entropies of the sequence. There are only a few results of this type for non-compact cases since both mass and entropy can escape. In this talk I'll describe how this can happen in the context of countable Markov shifts and give continuity results recently proved with Godofredo Iommi and Anibal Velozo.

**Weisheng Wu**

Title: On unstable entropies of partially hyperbolic diffeomorphisms

Abstract: Given a partially hyperbolic diffeomorphism, we introduce the notions of unstable metric entropy and unstable topological entropy. These notions coincide with the entropy defined by Ledrappier-Young and unstable volume growth rate considered by Hua-Saghin-Xia respectively. A variational principle can be established which states that unstable topological entropy is the supremum of unstable metric entropy taken over all invariant measures. Other good properties such as upper semi-continuity of unstable entropy function can be obtained. We also discuss relevant topics, including Bowen unstable entropy for partially hyperbolic diffeomorphisms; preimage entropy and folding entropy; and unstable entropy for nonuniformly hyperbolic diffeomorphisms.

**Leiye Xu**

Title: Ergodic optimization theory for typical maps and flows

Abstract: Ergodic optimization theory mainly studies the problems relating to minimizing (or maximum) orbits, minimizing (or maximum) invariant measures and minimizing (or maximum) ergodic averages. In this talk, we mainly introduce ergodic optimization problems of a class of dynamical systems. We show that for typical maps (includes Axiom A attractors, Anosov diffeomorphisms and uniformly expanding maps) or typical flows (Axiom A flows), the minimizing (or maximum) measures of generic  $H^k$ -order or  $C^1$  observables are unique and supported on a periodic orbit.

**Jinxin Xue**

Title: Rigidity of a class of abelian-by-cyclic group acting on torus.

Abstract: In this talk, we explain our work on the global rigidity of a class of abelian-by-cyclic group acting on torus. A prototype of such an action is generated by the Arnold cat map on the 2-torus together with a horizontal translation and a vertical translation. We give a complete classification of such actions up to both topological conjugacy and smooth conjugacy in the 2D case under certain genericity assumptions. We also obtain results in the higher dimensional case. This talk is based on joint work with Sebastian Hurtado and Amie Wilkinson.

**Zhiyuan Zhang (Monday)**

Title: On the set of pseudo-rotations on annulus

Abstract: A homeomorphism of the annulus without any periodic point is called a pseudo-rotation. Each pseudo-rotation has a unique rotation number in  $\mathbb{R}/\mathbb{Z}$ . We show that for a Baire generic rotation number  $\alpha \in \mathbb{R}/\mathbb{Z}$ , the set of area preserving  $C^\infty$  pseudo-rotation of the annulus  $A$  with rotation number  $\alpha$  equals to the closure of the set of area preserving  $C^\infty$  pseudo-rotations which are smoothly conjugate to the rotation  $R_\alpha$ . As a corollary, a  $C^\infty$  generic area preserving pseudo-rotation of the annulus with a Baire generic rotation number  $\alpha$  is weakly mixing. This is a joint work with Barney Bramham.

**Qi Zhou**

Title: Anosov-Katok construction for quasiperiodic cocycles

Abstract: In this talk, we will talk about Anosov-Katok construction for quasiperiodic  $SL(2, \mathbb{R})$  cocycles, and its various dynamical applications, for example, the continuity of the Lyapunov exponent, the growth of the cocycles.