Holomorphic dynamics and related fields

Gu Lecture Hall, SCMS

	Monday (9.23)	Tuesday (9.24)	Wednesday (9.25)	Thursday (9.26)	Friday (9.27)
9:30-10:20	M. Shishikura	W. Bergweiler	D. Smania	Y. Wang	Y. Cheung
10:20-10:50	Tea Break	Tea Break	Tea Break (Group Photo)	Tea Break	Tea Break
10:50-11:40	J. Souto	G. Cui	Rivera-Letelier	G. Zhang	Y. Shen
Lunch					
14:30-15:20	H. Sumi	F. Rong	Z. Li	H. Inou	G. Levin
15:20-15:50	Tea Break	Tea Break	Tea Break	Tea Break	Tea Break
15:50-16:40	Y. Xiao	Y. Okuyama	F. Yang	H. Ye	X. Wang
16:50-17:40	J. Hu		W. Cui		
		Banquet (Yanyuan Hotel)			

Walter Bergweiler
Title: Lebesgue measure of Julia sets of entire functions
Abstract: The Julia set of an entire function is the set where the family of iterates fails to be normal. The escaping set is the set of points which tend to infinity under iteration. McMullen showed that these sets have positive measure for the sine and cosine function. Since then these results have been extended to various classes of functions. We give a new criterion implying that these sets have positive measure. For example, the results can be applied to Poincar ́e functions of certain polynomials.

Yitwah Cheung
Title: Ergodicity of foliations defined by a holomorphic 1-form
Abstract: The real part of a holomorphic 1-form on a closed Riemann surface defines a measured foliation in the sense of Thurston, called the vertical. Foliations in other directions are obtained applying the action of multiplication by a complex unit. It is well-known that the foliation is uniquely ergodic in almost every direction. In fact, the Hausdorff dimension of the set of nonergodic directions is at most 1/2. Within a certain one-parameter family of genus 2 branched double covers of the torus the Hausdorff dimension can only take the values 0 or 1/2, depending on a.condition on the continued fraction of the parameter discovered by Perez Marco in the context of the linearization problem. In this talk, I will explain how the Perez Marco condition is used to build large.sets of nonergodic directions.

Guizhen Cui
Title: Twist deformation of rational maps
Abstract: Given a geometrically finite rational map with invariant arcs in Fatou domains, we define a sequence of quasiconformal deformation of the rational map by repeated Dean twist. We show the sequence is convergent.

Weiwei Cui
Title: On totally ramified rational maps
Abstract: A rational map is totally ramified if the preimages of each critical value are critical points. If, in addition, the multiplicities at critical points corresponding to the same critical value are equal, then we say the map is regularly ramified. In this talk, we show that totally ramified maps need not be regularly ramified by constructing examples using combinatorial methods. Moreover, we also discuss an algorithm for counting totally ramified rational maps. This is a joint work with Jun Hu.

Jun Hu
Title: Dynamics of degree-3 rational maps with escaping critical points.

Abstract: In the talk, we show that for any rational map f of degree 3, if f has an attracting fixed point p and all critical points are attracted to p under the iteration off, then the Julia set J(f) is either a Cantor set or a connected set (and locally connected). We also give a criterion for the connected Julia set to be a Sierpinski curve. This is a joint work with Arkady Etkin.

Hiroyuki Inou

Title: An example of an infinitely renormalizable cubic polynomial and its combinatorial class

Abstract: We construct an infinitely renormalizable cubic polynomial with two distinct critical points such that the filled Julia set of every renormalization contains both of the critical points. We would also discuss its dynamical behavior and its combinatorial class.

Genadi Levin

Title: (Positive) transversality of hyperbolic and parabolic maps.

Abstract: We consider families of holomorphic maps defined on subsets of the complex plane, and show that the technique developed in [1] to treat unfolding of critical relations can also be used to deal with cases where the critical orbit converges to a hyperbolic attracting or a parabolic periodic orbit. As before this result applies to families of maps provided some lifting property holds. We prove that either the transversality holds, that is, in this setting, multiplier of a hyperbolic attracting periodic orbit depends univalently on the parameter and bifurcations at parabolic periodic points are generic, or one has persistency of no transversality ,i.e., of periodic orbits with a fixed multiplier. As an application, we show that periodic points on the real line do not disappear after born for unimodal families of real maps like $g(x)+c$ and $c g(x)$, from the real quadratic one to the real sin-family and families with the flat critical point.
Joint work with Weixiao Shen and Sebastian van Strien.

[1] G. Levin, W. Shen and S. van Strien, Monotonicity of entropy and positively oriented transversality for families of interval maps, arXiv: 1611.10056, 2016.
[2] G. Levin, W. Shen and S. van Strien, Transversality in the setting of hyperbolic and parabolic maps, arXiv: 1901.09941, 2019

Zhiqiang Li

Title: Ergodic theory for expanding Thurston maps

Abstract: Thurston maps are a class of branched covering maps on the 2-sphere that arose in W. Thurston's characterization of postcritically finite rational maps. By imposing a natural expansion condition, M. Bonk and D. Meyer investigated a subclass of Thurston maps known as expanding Thurston maps, which turned out to enjoy nice topological, metric, and dynamical properties. Contrary to what the name may suggest, these maps have very weak expansion properties. This talk will serve as an introduction

to expanding Thurston maps. We will introduce these maps with some motivation from their connection to other topics of mathematics. We will then focus on the ergodic properties, especially the thermodynamical formalism for such maps.

Yusuke Okuyama
Title: Quantitative approximation of Lyapunov exponentsand its applications to the dynamical moduli spaces
Abstract: We talk about a locally uniform quantitative approximation of the Lyapunov exponent of a rational function of degree >1 in terms of the truncated logarithms of the moduli of the multipliers of its periodic points, over an algebraically closed field of characteristic 0 and that is complete with respect to a non-trivial and possibly non-archimedean absolute value. This approximation formula has applications in complex and arithmetic geometry of the dynamical moduli spaces of rational functions. This talk is based on joint works with Thomas Gauthier and Gabriel Vigny.

Feng Rong
Title: On two rigidity theorems with dynamical flavor
Abstract: We will talk about two classical theorems with connection to the rigidity part of the Schwarz lemma, Cartan's uniqueness theorem and Burns-Krantz boundary rigidity theorem. In the first part, we will focus on Cartan's linearity theorem, whose original proof uses Cartan's uniqueness theorem. By introducing the so-called "resonance order", we present a more "intrincic" proof of Cartan's linearity theorem. In the second part, we first give a dynamical interpretation of Burns-Krantz boundary rigidity theorem and then present some new boundary rigidity theorems on strongly pseudoconvex domains, whose proof uses the iteration theory on taut manifolds and hyperbolic Riemann surfaces in an essential way.

Yuliang Shen
Title: Weil-Petersson Teichmuller space and dependence of Riemann mappings for Weil-Petersson curves
Abstract: The classical Riemann mapping theorem implies that there exists a so-called Riemann mapping which takes the upper half plane onto the left domain bounded by a Jordan curve in the extended complex plane. A basic problem is to study how a Riemann mapping depends on the corresponding Jordan curve. We are mainly concerned with those Jordan curves in the Weil-Petersson class, namely, the corresponding Riemann mappings can be quasiconformally extended to the whole plane with Beltrami coefficients being square integrable under the Poincare metric. We show how to endow the space of all normalized Weil-Petersson curves with a new real Hilbert manifold structure, which is topologically equivalent to the standard complex Hilbert manifold structure. This implies that an appropriately chosen Riemann

mapping depends continuously on a Weil-Petersson curve (and vice versa). This is a joint work with Wu Li.

Mitsuhiro Shishikura
Title: Fingers in the complex Arnold family
Abstract: We study the bifurcation of the complex Arnold family $f (z) = z + \alpha + \beta \sin z$, where z is a complex variable, β is a small real parameter and the bifurcation is studied in the complex α-plane. One can see a structure called ``fingers'' or ``tentacles'' in the parameters space. (They are also observed for H\' {e} non maps.) We analyze the parabolic bifurcation and explain how such fingers are formed. This is a work in progress joint with David Marti Pete (Polish Academy of Science).

Daniel Smania
Title: Besov spaces and ergodic theory.
Abstract: The ergodic theory of smooth expanding maps is quite well developed. We know a lot on statistical properties of Holder observables, as exponencial decay of correlations and central limit theorem in this setting. However, if we have weaker assumptions on either the regularity of the maps, the phase space or the observables, things can became easily more involving. For instance we would like to study the statistical properties of discontinuous/unbounded observables on Julia sets of hyperbolic rational functions. Using the method of atomic decomposition we develop a theory of Besov spaces under very weak assumptions that allows us to study Ruelle-Perron-Frobenius transfer operators in very general settings. In particular we can study the action of transfer operators on Sobolev spaces defined in hyperbolic Julia sets. Joint work with Alexander Arbieto (UFRJ-Brazil).

Juan Souto
Title: Bubbling of quasi-regular maps
Abstract: Quasi-regular maps, that is quasi-conformal maps without the assumption that they are local homeomorphisms, share many properties with holomorphic functions in one variable. I will explain how exploit these similarities to prove, for the space of quasi-regular maps between two given manifolds, of a version of Gromov's compactness theorem for pseudo-holomorphic curves. I will then explain how understanding the possible degenerations of quasi-regular maps allows us to prove the following extension of Schoen's conjecture: quasi-regular maps from the n-sphere to itself extend to harmonic maps from $(n+1) $-dimensional hyperbolic space to itself. This is joint work with Pekka Pankka.

Hiroki Sumi

Title: Classifcation of generic random holomorphic dynamical systems associated with analytic families of rational maps

Abstract: We consider random holomorphic dynamical systems (with multiplicative noise) and we show that generic such systems have some kind of nice properties and order. This result represents a kind of randomness (noise)-induced phenomenon. Also, we classsify such systems. For the preprint, see
H. Sumi, Negativity of Lyapunov Exponents and Convergence of Generic Random Polynomial Dynamical Systems and Random Relaxed Newton's Methods, \\ https://arxiv.org/abs/1608.05230.

Juan Rivera-Letelier

Title: Complex dynamics and the neighbor exclusion model on the Cayley tree

Abstract. A description of the neighbor exclusion model on an infinite regular tree, and its connections to complex dynamics and independent sets. We show that the phase transition of the pressure function is of infinite order.
This is a joint work with M. Sombra.

Sebastian van Strien

Title: Conjugacy classes of real Analytic maps: on a question of Avila-Lyubich-de Melo

Abstract: Avila-Lyubich-de Melo proved that the topological conjugacy classes of unimodal real-analytic maps are complex analytic manifolds, which laminate a neighbourhood of any such mapping without a neutral cycle. Their proof that the manifolds are complex analytic depends on the fact that they are codimension-one in the space of unimodal mappings.

In joint work with Trevor Clark, we show how to construct a “pruned polynomial-like mapping" associated to a real mapping. This gives a new complex extension of a real-analytic mapping. The additional structure provided by this extension, makes it possible to generalize this result of Avila-Lyubich-de Melo to interval mappings with several critical points. Thus we show that the conjugacy classes are complex analytic manifolds whose codimension is determined by the number of critical points.

Building on these ideas, we believe we can show that in the space of unimodal mappings with negative Schwarzian derivative, the conjugacy classes laminate a neighbourhood of every mapping.

Xiaoguang Wang

Title: Newton's methods for polynomials: a dynamical system viewpoint.

Abstract: Newton's method is probably the oldest and most famous iterative process to be found in mathematics. In this talk, we will study the Newton's methods for arbitrary polynomials, from a dynamical system viewpoint. We will show that the boundary of any immediate root basin for the Newton map of any polynomial is locally connected. This implies that the boundaries of all components of root basins, for all polynomials' Newton maps, from the viewpoint of topology, are tame. This is a joint work with Yongcheng Yin and Jinsong Zeng.

Yuefei Wang

Title: On dynamics of entire maps with symmetry

Abstract: We will talk about recent results on the dynamics of holomorphic maps with certain symmetries, including families of Baker's wandering maps, exponential maps and hyperbolic maps, cosine maps etc. Such a map either contains a dynamical ray in its Julia set or has a multi-connected and bounded wandering domain (i.e. a Baker wandering domain). Moreover, any connected compact set containing the point on the ray and its forward image point intersects its Julia set. This is a joint work with Chunlei Cao.

Yingqing Xiao

Title: Self-affine measures and Beurling dimension

Abstract: Recently, we study the Beurling dimension of Bessel spectra and frame spectra of self-affine measures on R^d. We obtain upper bound of the Beurling dimension of Bessel spectra. Under a technical condition, lower bound of the Beurling dimension of frame spectra is also attained. Moreover, we study a class of spectra of the Sierpinski-type self-affine measures and show that the exact upper bound of their Beurling dimension is the Hausdorff dimension of the general Sierpinski carpets. This is a joint work with Zhang Zhanqi.

Fei Yang

Title: Maps in Inou-Shishikura's class and applications

Abstract: About 10 years ago, Inou and Shishikura constructed an invariant class of holomorphic maps under the parabolic and near-parabolic renormalization operators. It turns out that this invariant class is very useful in settling many important conjectures on the dynamics of quadratic polynomials. However, it is also known that some rational maps and entire functions also belong to this class. In this talk we first give a list of such specific rational maps and entire functions and then discuss the corresponding applications.

Hexi Ye

Title: A uniform bound for the number of common preperiodic points

Abstract: Let f_c (z) =z^2+c a the family of quadratic polynomials, parameterized by the complex number c. It was known that for any two distinct parameters c_1 and c_2 over the complex numbers, there are only finitely many preperiodic points of f {c_1} and f_{c_2} in common. In this talk, we are going to show that there is a uniform bound for the number of common preperiodic points as c_1 and c_2 vary. Moreover, the uniform bound is effective. This is a joint work with Laura DeMarco and Holly Krieger.

Gaofei Zhang

Title: Topological characterization of rational maps with Siegel disks

Abstract:We extend Thurston's topological characterization theorem for postcritically finite rational maps to a class of rational maps which have a fixed bounded type Siegel disk. This is the first result in generalizing Thurston's theorem to geometrically infinit e rational maps.