THE NUMBER OF CRITICAL SUBGRAPHS IN K-CRITICAL GRAPHS

Online seminar

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Abstract: Gallai asked in 1984 if any k-critical graph on n vertices contains at least n distinct $(k-1)$-critical subgraphs. The answer is trivial for $k\leq 3$. Improving a result of Stiebitz, Abbott and Zhou proved in 1995 that for all $k\geq 4$, any k-critical graph contains $\Omega(n^{1/(k-1)})$ distinct $(k-1)$-critical subgraphs. Since then no progress had been made until very recently, Hare resolved the case $k=4$ by showing that any 4-critical graph on n vertices contains at least $(8n-29)/3$ odd cycles.

In this talk, we mainly focus on 4-critical graphs and develop some novel tools for counting cycles of specified parity. Our main result shows that any 4-critical graph on n vertices contains $\Omega(n^2)$ odd cycles, which is tight up to a constant factor by infinitely many graphs. As a crucial step, we prove the same bound for 3-connected non-bipartite graphs, which may be of independent interest. Using the tools, we also give a short solution to Gallai's problem when $k=4$. Moreover, we improve the longstanding lower bound of Abbott and Zhou t $\Omega(n^{1/(k-2)})$ for the general case $k\geq 5$. We will also discuss related problems on k-critical graphs.