## Week 8 (Oct.30 - Nov.3)

Topic: 2D Hydrodynamics and related issues

Workshop Room: Room 2201, Guanghua East Building, Fudan University

## Lecture Series Speaker: Dong Li

#### **Invited Speakers:**

Taoufik Hmidi (Université de Rennes I) Renjin Jiang (Center for Applied Mathematics of Tianjin University) Tianling Jin (Hong Kong University of Science and Technology) Joan Mateu (Autonomous University of Barcelona) Gerard Misiołek (University of Colorado) Hideyuki Miura (Tokyo Institute of Technology) Yifei Wu (Center for Applied Mathematics of Tianjin University) Tsuyoshi Yoneda (Tokyo Institute of Technology)

#### **Organizing Committee:**

Peter Constantin (Princeton University) Yoshikazu Giga (University of Tokyo) Hao Jia (University of Chicago) Carlos Kenig (University of Chicago) Zhen Lei (Fudan University) Fanghua Lin (Courant Institute of Mathematical Sciences) Gregory Seregin (University of Oxford) Vladimir Sverak (University of Minnesota) Edriss Titi (Texas A & M University) Sijue Wu (University of Michigan)

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Shanghai Center for Mathematical Sciences School of Mathematical Sciences, Fudan University

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Schedule

## 2017 Fall Program on Analysis of PDE (Sept. 11 – Dec. 2, 2017)

Week 8 (Oct.30-Nov.3)		
Topic: 2D Hydrodynamics and related issues		
Monday (October 30)	Room 2201, Guanghua East Building, Fudan University	
Morning Session		
9:30 - 9:35	Chair: Fanghua Lin	
9:35 – 10:25	Dong Li	
10:25 - 10:45	Tea Break	
10:45 - 10:50	Chair: Dong Li	
10:50 - 11:40	Yifei Wu	
Lunch Break		
Afternoon Session		
14:30 - 14:35	Chair: Yifei Wu	
14:35 – 15:25	Gerard Misiolek	
15:25 - 15:45	Tea Break	
15:45 - 15:50	Chair: Gerard Misiolek	
15:50 - 16:40	Tianling Jin	
Tuesday (October 31)	Room 2201, Guanghua East Building, Fudan University	
Morning Session		
9:30 - 9:35	Chair: Fanghua Lin	
9:35 - 10:25	Dong Li	
10:25 - 10:45	Tea Break and Group Photo	
10:45 - 10:50	Chair: Dong Li	
10:50 - 11:40	Taoufik Hmidi	
Lunch Break		

Afternoon Session		
14:30 - 14:35	Chair: Taoufik Hmidi	
14:35 – 15:25	Joan Mateu	
15:25 – 15:45	Tea Break	
15:45 – 15:50	Chair: -	
15:50 - 16:40	-	
Wednesday (November 1) Room 2201, Guanghua East Building, Fudan University		
Morning Session		
9:30 – 9:35	Chair: Zhen Lei	
9:35 - 10:25	Dong Li	
10:25 - 10:45	Tea Break	
10:45 - 10:50	Chair: Dong Li	
10:50 - 11:40	Renjin Jiang	
Lunch Break		
Afternoon Session		
14:30 - 14:35	Chair: -	
14:35 – 15:25	-	
15:25 – 15:45	-	
15:45 - 15:50	Chair: -	
15:50 - 16:40	-	
Thursday (November 2) Room 2201, Guanghua East Building, Fudan University		
Morning Session		
9:30 – 9:35	Chair: Hao Wu	
9:35 - 10:25	Dong Li	
10:25 - 10:45	Tea Break	
10:45 - 10:50	Chair: Dong Li	
10:50 - 11:40	Hideyuki Miura	
Lunch Break		
Afternoon Session		

14:30 - 14:35	Chair: -	
14:35 – 15:25	-	
15:25 - 15:45	-	
15:45 - 15:50	Chair: -	
15:50 - 16:40	-	
Friday (November 3)	Room 2201, Guanghua East Building, Fudan University	
Morning Session		
9:30 - 9:35	Chair: Peng Qu	
9:35 - 10:25	Dong Li	
10:25 - 10:45	Tea Break	
10:45 - 10:50	Chair: Dong Li	
10:50 - 11:40	Tsuyoshi Yoneda	
Lunch Break		
Afternoon Session		
14:30 - 14:35	Chair: -	
14:35 – 15:25	-	
15:25 - 15:45	-	
15:45 - 15:50	Chair: -	
15:50 - 16:40	-	

## 2017 Fall Program on Analysis of PDE

Week 8 (Oct.30 - Nov.3)

Topic: 2D Hydrodynamics and related issues

#### **Titles and Abstracts:**

#### Speaker: Taoufik Hmidi

Title: Dynamics of the one-fold symmetric patches for the aggregation equation and collapse to singular measure

Abstract: We shall consider the two-dimensional aggregation equation with Newtonian potential and where the initial data is in the patch form with an axis of symmetry. We discuss in particular the local/ global existence of the associated graph in different function spaces like H\"{0}lder and Dini spaces. We shall also analyse the asymptotic behavior for large time and prove the convergence in a suitable way to a singular measure supported in a union of disjoint segments. As we shall see, the density can be recovered from the global dynamics of the graph. This is a joint work with Dong Li.

#### **Speaker: Renjin Jiang**

#### Title: Transport equation and flow for critical vector fields

**Abstract:** In this talk, I will provide sharp conditions for the divergence of vector fields such that the transport equation is well-posed is L^\infty and the flow exists with suitable density. Examples will also be provided to show the sharpness of the condition. This is a joint work with A. Clop, J. Mateu and J. Orobitg.

#### **Speaker: Tianling Jin**

#### Title: On the best constants and minimizers for the fractional Sobolev inequality in domains

**Abstract:** We consider a version of the fractional Sobolev inequality in domains and study whether the best constant in this inequality is attained. For the half-space and a large class of bounded domains we show that a minimizer exists, which is in contrast to the classical Sobolev inequalities in domains.

#### **Speaker: Joan Mateu**

#### **Title: V-states and dislocations**

**Abstract:** I will talk about two different problems: 2-D rotating vortex patches and dislocations. I will present some results obtained about rotating patches for the 2-D Euler equation. We will see how the techniques used to solve this problem has been useful to solve a problem of minimization of energy in the planar case. We also present a recent result of the minimization of a given energy in the 3-D case.

#### Speaker: Gerard Misiołek

#### Title: Regularity properties of the solution map of the incompressible Euler equations

**Abstract:** I will present a simple counterexample to continuity of the solution map in the Holder  $C^{1,\alpha}$  space and argue that a natural setting for well-posedness of the Euler equations in the sense of Hadamard is its separable subspace  $c^{1,\alpha}$ . The latter is partly based on the Lagrangian approach and, time permitting, I will describe some recent results on the solution map in the Lagrangian setting.

#### Speaker: Hideyuki Miura

#### Title: On Ukai-type solution formula for the Stokes system in a domain with graph boundary

Abstract: We consider the space of solenoidal vector fields in an unbounded domain whose boundary is given as a Lipschitz graph. It is shown that, under suitable functional setting, the space of solenoidal vector fields is isomorphic to the n - 1 product space of the space of scalar functions. As an application, we introduce a systematic reduction of the equations describing the motion of incompressible flows. This gives a new perspective of the derivation of Ukai's solution formula for the Stokes equations in the half space, and provides a key step for the generalization of his approach to the Stokes semigroup in the case of the domain with the curved boundary. This is joint work with Yasunori Maekawa.

Speaker: Dong Li Title: Abstract:

#### Speaker: Yifei Wu

Title: Instability of the standing waves for the nonlinear Klein-Gordon equations in one dimension Abstract: The nonlinear Klein-Gordon equation has the standing wave solutions  $u_\omega=e^{i\circ}$ t}\phi\_{\omega}\$ in the L2-subcritical case, with the frequency  $\omega(in(-1,1))$ . It was proved by Shatah (1983), and Shatah, Strauss (1985) that there exists a critical frequency  $\omega_c(in (0,1))$  such that the standing waves solution  $u_\omega$  is orbitally stable when  $\omega_c<|omega|<1$ , and orbitally unstable when  $|\circomega|<\circmega_c$ . Further, the critical case  $|\circomega|=|\circomega_c$  in the high dimension  $d\ge 2$  was considered by Ohta, Todorova (2007), who proved that it is strongly unstable, by using the virial identities and the radial Sobolev inequality. The one dimension problem was left after then. In this paper, we consider the onedimension problem and prove that it is orbitally unstable when  $|\circomega|=|\circomega_c$ .

#### Speaker: Tsuyoshi Yoneda

# Title: A remark on trajectory behavior along the axis for the axisymmetric Euler equations with large uniform rotation

**Abstract:** The dynamics along the particle trajectories for the 3D axisymmetric Euler equations with large uniform rotation are considered. It is shown that any initial particle with non-zero velocity never touch any stagnation point

in finite time provided by large uniform rotation. In the proof, Frenet-Serret formulas and orthonormal moving frame are essentially used. This mathematical study is closely related to a physical phenomenon: vortex breakdown, and this is a joint work a physicist: Doctor Kento Yamada whose study field is the "vortex breakdown".

### **Participants:**

Yuan Cai (Fudan University) Tuowei Chen (Fudan University) Xiufang Cui (Fudan University) Taoufik Hmidi (Université de Rennes I) Bobo Hua (Fudan University) Renjin Jiang (Center for Applied Mathematics of Tianjin University) Tianling Jin (Hong Kong University of Science and Technology) Zhentao Jin (Fudan University) Zhen Lei (Fudan University) Dong Li (University of British Columbia) Fanghua Lin (Courant Institute) Guowei Liu (Shanghai Jiaotong University) Song Liu (Chinese Academy of Science) Xiang Luo (University of Science and Technology of China) Joan Mateu (Autonomous University of Barcelona) Gerard Misiołek (University of Colorado) Hideyuki Miura (Tokyo Institute of Technology) Yun Pu (Fudan University) Peng Qu (Fudan University) Jiajun Tong (Courant Institute) Chenmu Wang (Fudan University) Xiaoming Wang (Fudan University & Shanghai Center for Mathematical Sciences) Yanyan Wang (Fudan University) Yucong Wang (Xiamen University) Hao Wu (Fudan University) Sijue Wu (University of Michigan) Xiaochun Wu (Fudan University) Yifei Wu (Center for Applied Mathematics of Tianjin University) Tsuyoshi Yoneda (Tokyo Institute of Technology) Meng Yuan (Nanjing University) Lan Zhang (Wuhan University) Jing Zhang (Fudan University) Na Zhao (Fudan University) Yi Zhou (Fudan University)