

# **The Eleventh Pacific Rim Geometry Conference**

## December 10-13, 2013

# Room 2001, Guanghua East Building, Fudan University, Shanghai

 $k_3 = hf(x_{i-1} + \frac{h}{2}, y_{i-1} + \frac{k_2}{2})$ Scientific Committee Qing Ding, Yuxin Dong, Jixiang Fu, Jun Li, Weiping Zhang

Sponsored by Shanghai Center for Mathematical Sciences, School of Mathematical Sciences, Fudan University

Schedule

### Tuesday, December 10

8:30-9:20	Registration			
9:30-9:40	Opening			
9:40-10:40	Rick Schoen	Stanford Univ.	Some geometric problems arising in general	
			relativity	
Tea Break				
11:00-12:00	Xiping Zhu	Sun Yat-sen Univ.	Lipschitz continuity of harmonic maps between	
			Alexandrov spaces	
Lunch Time 15th floor café or dinning hall <sup>1</sup>				
2:30-3:30pm	Mathai	Univ. of Adelaide	Quantizing Group Actions	
	Varghese			
Group Photo South entrance of Guang Hua building				
4:30-5:30pm	Martin	Waseda Univ.	The tt*-Toda equations	
	Guest			
6:15pm Banquet Shunfeng Restaurant, 5th floor of Oriental Mansion, No 2500 Siping RD				

### Wednesday, December 11

9:30-10:30	Ngaiming	Univ. of Hong	On the Zariski closure of the image of an algebraic	
	Mok	Kong	set on a bounded symmetric domain under the	
			uniformization map	
Tea Break				
11:00-12:00	Xiangyu	Chinese	L <sup>2</sup> extension and multiplier ideal sheaf	
	Zhou	Academy of		
		Sciences		
Lunch Time 15th floor café or dinning hall				
2:30-3:30pm	Kefeng Liu	UC Los Angeles	Curvatures of direct image sheaves of vector	
			bundles and applications	
Tea Break				
4:00-5:00pm	Haizhong Li	Tsinghua	Embedded constant mean curvature tori in the	
		University	three-sphere	
5:45pm	Dinner (Invited Guests) Yanyuan Hotel, No 270 Zhengtong Rd (near Guoding Rd)			

<sup>&</sup>lt;sup>1</sup> Those with meal cards please go to the university dinning hall for lunch; those without meal cards will have box lunch at 15th floor cafe.

## Thursday, December 12

9:30-10:30	Shu-Cheng	Taiwan Univ.	On the uniformization theorem in a strictly		
	Chang		pseudoconvex CR-manifold		
Tea Break					
11:00-12:00	Qing-Ming	Fukuoka Univ.	Complete self-shrinkers of the mean curvature flow		
	Cheng				
Lunch Time 15th floor café or dinning hall					
2:30-3:30pm	Yuan-Long	Fudan Univ.	Some new aspects of the Bernstein Theorem		
	Xin				
Tea Break					
4:00-5:00pm	Qun Chen	Wuhan Univ.	Some results on V-harmonic maps and their		
			applications		
5:45pm	Dinner (Invited Guests) Yanyuan Hotel, No 270 Zhengtong Rd (near Guoding Rd)				

## Friday, December 13

9:30-10:30	Gang Tian	BICMR,	Partial C <sup>0</sup> -estimate and its applications		
		Princeton Univ.			
Tea Break					
11:00-12:00	Zizhou Tang	Beijing Normal	Recent works on isoparametric foliation		
		Univ.			
Lunch Time 15th floor café or dinning hall					
1:30-2:30pm	Conan	Chinese Univ. of	SYZ mirror symmetry		
	Leung	Hong Kong			
Tea Break					
2:45-3:45pm	Jiayu Li	USTC	Asymptotic behavior of the Yang-Mills-Higgs flow		
Tea Break					
4:00-5:00pm	Zhongmin	IUPUI	On Einstein Metrics in Finsler Geometry		
	Shen				

#### ABSTRACT

Speaker: Shu-Cheng Chang (Taiwan University)

**Title:** On the uniformization theorem in a strictly pseudoconvex CR manifold

**Abstract:** In this talk, via CR Calabi-Yau Theorem and Mok-Siu-Yau type estimate, we first affirm a partial answer of CR Frankel conjecture in a closed strictly pseudoconvex CR (2n + 1)-manifold with positive pseudohermitian bisectional curvature for  $n \ge 2$ . Secondly, by applying a linear trace Li-Yau-Hamilton inequality for a positive (1, 1)-form solution of the CR Hodge-Laplace heat equation and monotonicity of the heat equation deformation, we obtain an optimal gap theorem for a complete strictly pseudocovex CR (2n+1)-manifold with nonnegative pseudohermitian bisectional curvature and vanishing torsion.

**Speaker:** Qun Chen (Wuhan University)

Title: Some results on V-harmonic maps and their applications

**Abstract:** V-harmonic map is a generalization of harmonic maps in Hermitian, Weyl, affine and Finsler geometry. In this talk, we will introduce maximum principles, existence and Liouville theorems for V-harmonic maps and their applications. These are joint works with J. Jost, G.F. Wang and H.B. Qiu, respectively.

Speaker: Qing-Ming Cheng (Fukuoka University)

Title: Complete self-shrinkers of the mean curvature flow

**Abstract:** In this talk, I would like to consider complete self-shrinkers of the mean curvature flow. I shall talk about classifications of complete self-shrinkers and discuss the rigidity results of complete self-shrinkers.

**Speaker:** Martin Guest (Waseda University)

**Title:** The  $tt^*$ -Toda equations

Abstract: The  $tt^*$  (topological-antitopological fusion) equations were introduced by Cecotti and Vafa in the early 1990s. Solutions can be interpreted as pluriharmonic maps. Certain special solutions can be interpreted as deformations of quantum field theories, Frobenius manifolds with real structure, or variations of noncommutative Hodge structures. I will describe some solutions in a special case, the  $tt^*$ -Toda equation, which is a generalization of the third Painleve equation. This is joint work with Alexander Its and Chang-Shou Lin. **Speaker:** Conan Leung (Chinese University of Hong Kong)

Title: SYZ mirror symmetry

Speaker: Haizhong Li (Tsinghua University)

Title: Embedded constant mean curvature tori in the three-sphere

Abstract: This a joint work with Ben Andrews. The minimal surface is the surface with constant mean curvature zero. It was conjectured by H. B. Lawson in 1970's that the only embedded minimal torus in three-sphere is the Clifford torus. In 1980's, U. Pinkall and I. Sterling conjectured that embedded tori with CMC in three-sphere are surfaces of revolution. At March of 2012, Simon Brendle of Stanford University solved the Lawson conjecture. At April of 2012, Ben Andrews and Haizhong Li gave a complete classification of CMC embedded tori in the three-sphere. When the constant mean curvature is equal to 0 or  $\pm \frac{1}{\sqrt{3}}$ , the only embedded torus is the Clifford torus. For other values of the mean curvature, there exists embedded torus which is not the Clifford torus, Ben Andrews and Haizhong Li gave a complete description of such surfaces. As a Corollary, Ben Andrews and Haizhong Li Theorem have solved the famous Pinkall-Sterling conjecture.

Speaker: Jiayu Li (University of Science and Technology of China)

Title: Asymptotic behavior of the Yang-Mills-Higgs flow

**Abstract:** In this talk, we will introduce our recent work on the existence of approximate Hermitian-Einstein structures on semi-stable Higgs bundles, and the asymptotic behavior of the Yang-Mills-Higgs flow for Higgs pairs at infinity. This is a joint work with Xi Zhang.

**Speaker:** Kefeng Liu (University of California, Los Angeles)

Title: Curvatures of direct image sheaves of vector bundles and applications

Speaker: Ngaiming Mok (University of Hong Kong)

**Title:** On the Zariski closure of the image of an algebraic set on a bounded symmetric domain under the uniformization map

Abstract: The study of tautological foliations arising from minimal rational curves on uniruled projective manifolds (e.g. Fano manifolds) is crucial for the understanding of the geometry of such manifolds. In the special case of Hermitian symmetric manifolds M of the compact type it is natural to consider an analogous dual situation, viz., tautological foliations arising from minimal disks on finite-volume quotients  $X := \Omega/\Gamma$  of bounded symmetric domains  $\Omega \subset \subset \mathbb{C}^n \subset M$ , where the inclusions are the Harish-Chandra and Borel embeddings. Here by a minimal disk  $\Omega$  on X we mean the image of a minimal disk on under the

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uniformization map  $\pi: \Omega \to X$ . This situation and its generalizations arise naturally in the study of Zariski closures of totally geodesic complex submanifolds of Shimura varieties. More generally, letting  $S \subset \mathbb{C}^n$  be an affine-algebraic set, and writing  $Z \subset X$  for the Zariski closure of  $\pi(S \cap \Omega)$  in X, we had conjectured that  $Z \subset X$  is totally geodesic. The latter conjecture turns out to be equivalent to the Hyperbolic AxLindemann Conjecture in Functional Transcendence Theory, which had been formulated for arithmetic lattices  $\Gamma \subset Aut(\Omega)$  and very recently established by Klingler-Ullmo-Yafaev by using o-minimality in Model Theory. For this approach the arithmeticity of the lattice is essential. We establish the total geodesy of  $Z \subset X$  without assuming arithmeticity of  $\Gamma$ . The proof is based on methods of Kähler geometry in the study of asymptotic properties of algebraic subsets of  $\Omega$  as they exit the boundary  $\partial\Omega$ , methods which we had introduced earlier to solve the problem in the rank-1 case, and by the construction of irreducible components  $\tilde{Z} \subset \Omega$  as algebraic subsets. As an important intermediate step we show that  $Z \subset X$  is uniruled by holomorphic geodesic curves.

**Speaker:** Rick Schoen (Stanford University)

Title: Some geometric problems arising in general relativity

**Abstract:** We will discuss questions related to the constraint equations of general relativity and the theory of marginally outer trapped surfaces (MOTS). MOTS are a natural generalization of stable minimal hypersurfaces, and their existence or non-existence in a given initial data set has significance for the evolution and also for the geometry of the data. We will introduce these concepts and describe recent work in the area.

Speaker: Zhongmin Shen (Indiana University - Purdue University Indianapolis)

Title: On Einstein Metrics in Finsler Geometry

**Abstract:** Finsler metrics are just regular metrics without quadratic restriction. The notion of Ricci curvature in Riemannian geometry can be extended to Finsler metrics. It is a natural problem to study and characterize Finsler metrics with isotropic Ricci curvature (Einstein-Finsler metrics). Recently, some global rigidity theorems on Einstein-Finsler manifolds have been established. Many interesting Einstein-Finsler metrics have been a constructed. In this talk I will give a brief survey on Einstein-Finsler metrics and related topics.

Speaker: Zizhou Tang (Beijing Normal University)

Title: Recent works on isoparametric foliation

**Speaker:** Gang Tian (Beijing International Center for Mathematical Research, Princeton University)

**Title:** Partial  $C^0$ -estimate and its applications

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**Speaker:** Mathai Varghese (University of Adelaide)

Title: Quantizing Group Actions

Abstract: The "quantization commutes with reduction" problem was solved for compact groups acting on compact symplectic manifolds by Meinrenken, with Tian-Zhang giving a direct analytic proof in the late 1990's. Since then, generalizations have been obtained to cases where only the group, or the orbit space of the action, is compact. The main result which I will discuss is a generalization to settings where the group, the manifold and the orbit space may all be noncompact, as long as the symplectic reduction at zero is compact. The method used builds on the Tian-Zhang approach in the compact case. This is joint work with Peter Hochs.

Speaker: Yuan-Long Xin (Fudan University)

Title: Some new aspects of the Bernstein Theorem

**Abstract:** There are various generalizations of the well-known Bernstein theorem for minimal graphs in  $\mathbb{R}^3$  over the past decades. We will review some recent developments for minimal parametric hypersurfaces in Euclidean space, higher codimensional generalizations for Moser's weak version of Bernstein's theorem (so-called Lawson-Osserman problem), as well as the Bernstein type theorems for minimal hypersurfaces in general Riemannian manifolds with non-negative Ricci curvature.

Speaker: Xiangyu Zhou (Chinese Academy of Sciences)

**Title:**  $L^2$  extension and multiplier ideal sheaf

Speaker: Xi-Ping Zhu (Sun Yat-sen University)

Title: Lipschitz continuity of harmonic maps between Alexandrov spaces

**Abstract:** In 1997, J. Jost and F. H. Lin, independently proved that every energy minimizing harmonic map from an Alexandrov space with curvature bounded from below to an Alexandrov space with nonpositive curvature is locally Holder continuous. Meanwhile, F. H. Lin proposed a problem: whether the Holder continuity can be improved to Lipschitz continuity? J. Jost also asked a similar problem about Lipschitz regularity of harmonic maps between singular spaces in 1998. In this talk, we give an affirmative answer. This is a joint work with Hui-Chun Zhang.

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### Venue of the Conference:

2001, East Guanghua Building, Fudan University (No 220 Handan Road)

 $b_i - (\sum_i a_{ii} x)$ 

Lodging: speakers and participants are staying in the following three hotels:

- 1. Yanyuan Hotel (燕园宾馆), No 270 Zhengtong Road (Near Guoding Rd)
- 2. Zhengda Hotel (正大卿云宾馆), No 220 Handan Road, Fudan campus
- 3. Crowne Plaza Hotel (复旦皇冠假日酒店), No 199 Handan Road (Near Guoquan Rd)

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