Coprime Mappings and Lonely Runners

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Zoom meeting ID: 898 1138 0689 Password: 121323
Link: https://zoom.us/j/89811380689

Abstract:

The lonely runner conjecture can be stated as follows: for any n positive integers $v_1 < v_2 < \ldots < v_n$ there exists a real number t such that each $v_i t$ is at least $1/(n+1)$ away from the nearest integer. In this paper we prove that this is true if $v_n < (2-\varepsilon)n$. This is an approximate version of a natural next step for the study of the lonely runner conjecture suggested by Tao.

The key ingredient in our proof is a result on coprime mappings. Let A and B be sets of integers. A bijection $f : A \to B$ is a coprime mapping if a and $f(a)$ are coprime for every $a \in A$. We show that if $A, B \subset [n]$ are intervals of length $2m$ where $m = \exp(\Omega((\log\log n)^2))$ then there exists a coprime mapping from A to B. This is based on joint work with Tom Bohman.