MINIMAL MASS BLOW-UP SOLUTIONS FOR THE L^2 CRITICAL NLS WITH THE DELTA POTENTIAL FOR THE RADIAL DATA IN ONE DIMENSION

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Abstract: We consider the L^2 -critical nonlinear Schrödinger equation (NLS) with the delta potential $i\partial_t u + \partial_x^2 u + \mu \delta u + |u|^4 u = 0$, $t \in R, x \in R$, where $\mu \in R$, and δ is the Dirac delta distribution at $x = 0$ .Local well-posedness theory together with sharp Gagliardo-Nirenberg inequality and the conservation laws of mass and energy implies that the solution with mass less than $\|Q\|_2$ is global existence in $H^1(R)$, where Q is the ground state of the L^2-critical NLS without the delta potential (i.e. $\mu = 0$).

We are interested in the dynamics of the solution with threshold mass $\|u_0\|_2 = \|Q\|_2$ in $H^1(R)$. First, for the case $\mu = 0$), such

blow-up solution exists due to the pseudo-conformal symmetry of the equation, and is unique up to the symmetries of the equation in $H^1(R)$ from \cite{Me93:NLS:mini sol} (see also \cite{HmKe05:NLS:mini blp}), and recently in $L^2(R)$ from \cite{Dod:NLS:L2thrh1}. Second, for the case $\mu < 0$, simple variational argument with the conservation laws of mass and energy implies that radial solutions with threshold mass exist globally in $H^1(R)$. Last, for the case $\mu > 0$, we show the existence of radial threshold solutions with blow-up speed determined by the sign (i.e. $\mu > 0$) of the delta potential perturbation since the refined blow-up profile to the rescaled equation is stable in a precise sense. The key ingredients here including the Energy-Morawetz argument and compactness method as well as the standard modulation analysis. It is a joint work with Xingdong Tang.