SPECIAL CUBE COMPLEXES AND 3-MANIFOLD GROUPS

PART I

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RAAG

$\Gamma = (V\ E)$

$A(\Gamma) = \langle x_i : i \in V \mid [x_i\ x_j] = 1 : \{i . j\} \in E \rangle$

$ART(\Gamma)$:

Salvetti cplx

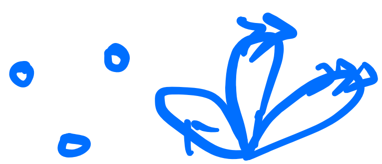
0- cube $\quad *$

1 - cubes $\longleftrightarrow x_i$

2 - cubes $\longleftrightarrow [x_i\ x_j]$

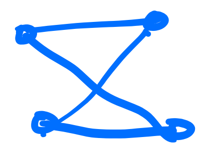
...

k - cubes $\longleftrightarrow \{x_{i_1} \ldots x_{i_k}\}$ p/w commut.



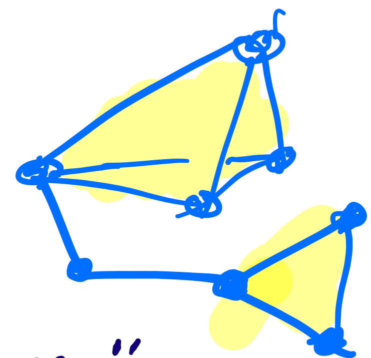
F_3

$\mathbb{Z}^2$



$F_2 \times F_2$

- ART(Γ)
  - finite (if Γ finite)
  - lk(*)



"any seeing simplex of dim ≥ 2 is filled up"

↳ "flag"

"a cube cplx with all vertex links flag"

↳ "nonpositively curved" (npc)

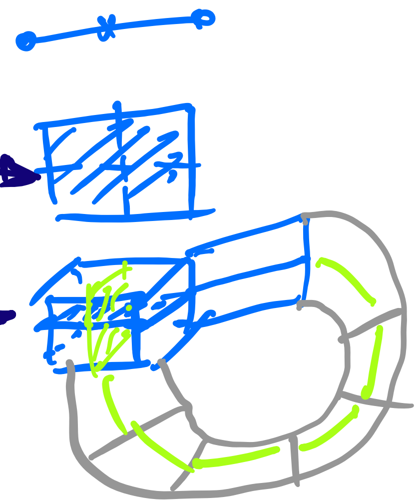
• $A(\Gamma) \curvearrowright \widetilde{ART(\Gamma)}$

"properly discontinuous"

(actually free here)

↳ "CAT(0)" $=$ npc + 1-conn.

• In $ART(\Gamma)$, "midcube"



"hyperplane"

h/p have no self-intersection or some "pathologies".

nicely positioned.

- Motivations from 3D topology

Thurston — 1982

- Virtual Haken Conjecture (VHC)
- vir. fibering conjecture (VFC)

Agol — 2012

Agol's RFRS criterion (2008):

M^3 cpt, or'ble, $\partial = \emptyset$ or tori.

$\pi_1 M$ RFRS $\Rightarrow$ M virtually fibers.

"Residually finite rationally solvable"

G RFRS $:\Leftrightarrow$

f.g. $\exists\ G_0 \geq_{f.i.} G_1 \geq_{f.i.} G_2 \geq \cdots$, $G_0 = G$

s.t. • $\bigcap_i G_i = \{1\}$

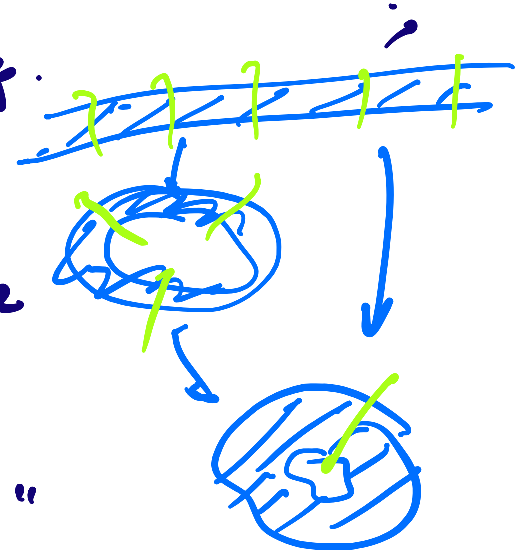
• $\mathrm{Ker}(G_i \to G_i^{fab}) \leq G_{i+1}$

$G_i^{fab} \hookrightarrow H_1(G_i;\mathbb{Z})/\mathrm{tors}$

Rmk:

(1) can further require "$G_i \trianglelefteq G$".

(2) topological meaning.



"the sequence can arise by taking finite cyclic covers dual to nonseparating surfaces"

- Agol: f.g subgps of RAAGs / RACGs are RFRS.
- Wise 2012: VHC $\Rightarrow$ VFC.
- Agol 2012: VHC ✓

- Prior to that

Bergeron–Haglund–Wise (2011)

"hyperplane sections in arithmetic hyperbolic manifolds"

- Haglund–Wise (2008)

"Special Cube Complexes"

TBC