SPECIAL CUBE COMPLEXES AND 3-MANIFOLD GROUPS

PART II

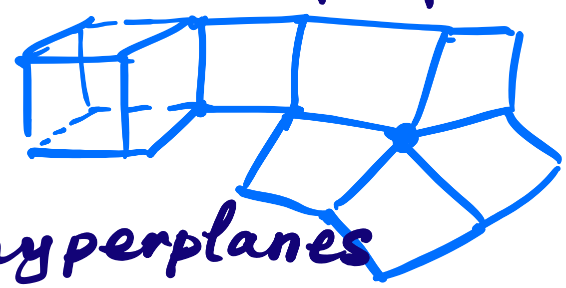
Yi Liu (BICMR)

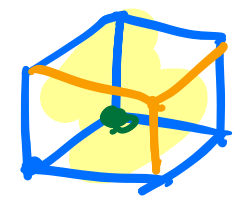
Shanghai (zoom), August 2022

Recall:

- A cube cplx is made up of cubes

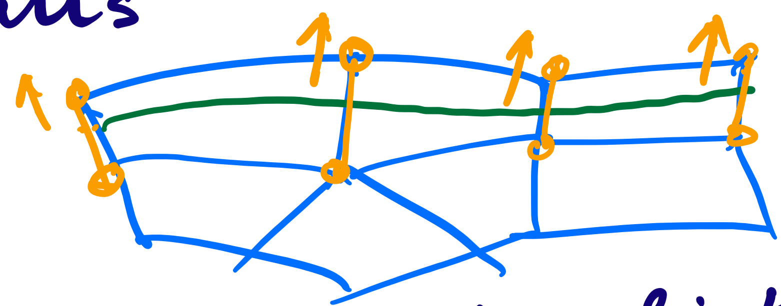
  $[-1,1]^n$





- Midcubes form hyperplanes

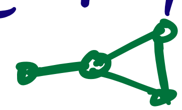
  "walls"



- npc $\iff$ vertex links are all flag.

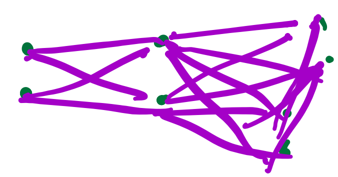
  CAT(0) $\iff$ npc + 1-conn.

$\Gamma = (V, E)$



$ART(\Gamma)$ $lk(*)$

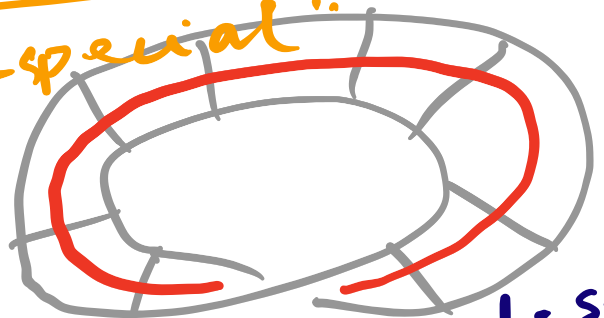
$A(\Gamma) \curvearrowright ART(\Gamma)$



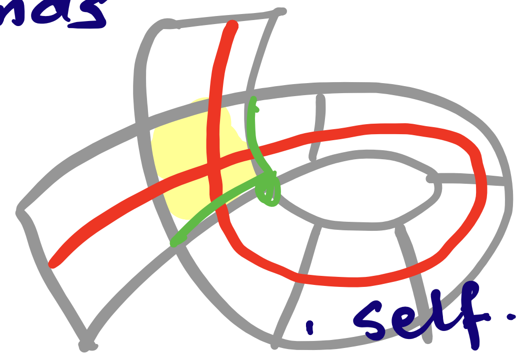
· special ⟺ no pathologies of 4 kinds

"A-special"

"C-special"



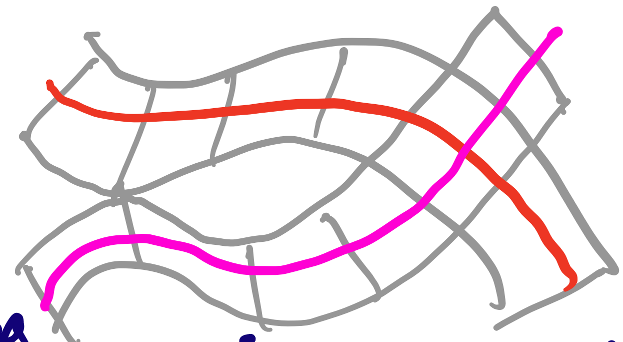
· 1-sided



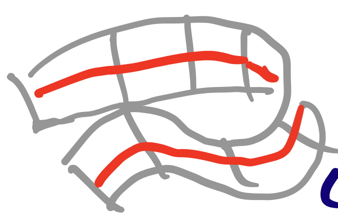
· self-intersecting

"A special cube cplx is"...

· self-osculating (indirect)



· interosculating

[ (direct)]

Haglund — Wise

- What does a special cub cplx have to do with RAAGs?
- How to verify/characterize virtual specialness?

THEOREM (HW): X npc + special $\Rightarrow$ $\pi_1 X \hookrightarrow$ a RAAG.

[Γ : vertices $\leftrightarrow$ hyperplanes in X
edges $\leftrightarrow$ intersecting pairs of h/ps.

$A(\Gamma) \curvearrowright \widetilde{ART(\Gamma)}$. $ART(\Gamma)$.

Since h/ps in X are 2-sided.
can fix coorientation (sides).

$$\tau_A : X \longrightarrow ART(\Gamma)$$

a combinatorial map.

vertices $\mapsto$ $*$

oriented edges $\mapsto$ "the dual h/p edge"

squares $\mapsto$ ✓

higher cubes $\mapsto$ ✓

Observe: if τ_A is locally isometric then $\tau_{A\#} : \pi_1 X \hookrightarrow A(\Gamma)$.

specialness $\Rightarrow$ local isometry.

For instance,

. virtual specialization

(as a program)

e.g.

Given a group G,

a procedure called "Sageev construction"

(1) Try to "cubulate G"

$G \curvearrowright X$ CAT(0) cube cplx.

properly discont.

In closed hyp. 3-mfd. the codim-1 subgps needed are provided by the Kahn-Markovic QF subsurfaces.

(2) Try to "virtually specialize the cubulation"

$\dot{G} \leq_{f.i.} G$, such that

$\dot{G} \curvearrowright X$ freely

special

$X/\dot{G}$ special

If so, then G is VS.

For (2).

- Use the characterizations of HW.
  e.g. Bergeron–Haglund–Wise
- Use Wise's QVH

hyp. + QVH ⇒ VS.

$A *_C B$. $A *_C$

TBC