LATTICE APPROXIMATIONS TO NONLINEAR DISPERSIVE EQUATIONS

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Time: Fri, Dec 6th, 09:00 - 10:00
Venue: Room 110, SCMS

Abstract:

Lattice models play a pivotal role in the investigation of microscopic multi-particle systems, with their continuum limits forming the foundation of macroscopic effective theory. These models have found broad applications in condensed matter physics, numerical analysis, and PDE.

In this talk, I will present our recent work on the continuum limits of some lattice models to the corresponding nonlinear dispersive equations. Using the integrable Ablowitz--Ladik system as a prototype, we establish that solutions of this discrete model converge to solutions of either the cubic nonlinear Schr\"odinger equation (NLS) or the modified Korteweg–de Vries equation (mKdV) in certain limiting regimes. Notably, we consider white-noise-like initial data which excites Fourier modes throughout the circle, and demonstrate convergence to a system of NLS/mKdV. This result suggests that a sole continuum equation may not suffice to encapsulate the lattice dynamics in such a low-regularity setting reminiscent of the thermal equilibrium state.

Additionally, I will outline the framework of our proof and discuss how it has been extended to address more general lattice approximations to dispersive PDEs, which further gives insight into constructing dynamics for the Landau–Lifschitz spin model in its Gibbs state.