Random Turán and counting results for general position sets over finite fields

Speaker: Jiaxi Nie
Shanghai Center for Mathematical Sciences

Time: Wed, Nov. 22nd, 16:00-16:30
Venue: Room 102, SCMS

Abstract:

Let $\alpha(\mathbb{F}_q^d,p)$ denote the maximum size of a general position set in a p-random subset of $\mathbb{F}_q^d$.

We determine the order of magnitude of $\alpha(\mathbb{F}_q^2,p)$ up to polylogarithmic factors for all possible values of p, improving the previous best upper bounds obtained by Roche-Newton--Warren and Bhowmick--Roche-Newton. For $d \ge 3$ we prove upper bounds for $\alpha(\mathbb{F}_q^d,p)$ that are essentially tight within certain intervals of p.

We establish the upper bound $2^{(1+o(1))q}$ for the number of general position sets in $\mathbb{F}_q^d$, which matches the trivial lower bound 2^{q} asymptotically in the exponent. We also refine this counting result by proving an asymptotically tight (in the exponent) upper bound for the number of general position sets with fixed size. The latter result for $d=2$ improves a result of Roche-Newton--Warren.

Our proofs are grounded in the hypergraph container method, and additionally, for $d=2$ we also leverage the pseudorandomness of the point-line incidence bipartite graph of $\mathbb{F}_{q}^2$.