

DEGREE GROWTH OF ENDOMORPHISMS

Speaker: Fei Hu
Nanjing University

Time: Fri., Oct.16th, 09:30-11:00

Venue: Room 102, SCMS

Abstract: Let X be a normal projective variety of dimension $d \geq 2$, and let f be a surjective endomorphism of X . For an ample divisor H and $1 \leq k \leq d-1$, the degrees $\deg_k(f^n) := (f^n)^* H^k \cdot H^{d-k}$ satisfy $\deg_k(f^n) \sim n^{\mu_k} \lambda_k^n$, where λ_k is called the k -th dynamical degree of f . The dynamical degrees are log-concave: $\lambda_k^2 \geq \lambda_{k-1} \lambda_{k+1}$. We show that strict log-concavity at k forces $\mu_k = 0$, while equality implies $0 \leq 2\mu_k - \mu_{k-1} - \mu_{k+1} \leq 4$. As a consequence, we obtain the sharp upper bound $\mu_k \leq 2k(d-k)$, extending Dinh - Lin - Oguiso - Zhang's codimension-one bound $\mu_1 \leq 2(d-1)$ in the zero-entropy case to arbitrary codimension and entropy. For zero-entropy automorphisms, we also prove that the mixed orbit intersection function associated with a weak composition γ of d is a multivariate quasipolynomial of even total degree at most $d^2 - \sum_j \gamma_j^2$. The proofs use a Hodge - Lorentz framework combining the Hodge index property with positivity. This framework also yields a weaker form of the reverse Khovanskii - Teissier inequalities of Xiao and Jiang - Li. A key ingredient is a four-point inequality of Ptolemy type for mixed intersections of nef divisors. This is joint work with Chen Jiang.